Grade 10

Mathematics and Science

A Student and Family Guide
Dear Student and Parent:

The Texas Assessment of Knowledge and Skills (TAKS) is a comprehensive testing program for public school students in grades 3–11. TAKS replaces the Texas Assessment of Academic Skills (TAAS) and is designed to measure to what extent a student has learned, understood, and is able to apply the important concepts and skills expected at each tested grade level. In addition, the test can provide valuable feedback to students, parents, and schools about student progress from grade to grade.

Students are tested in mathematics in grades 3–11; reading in grades 3–9; writing in grades 4 and 7; English language arts in grades 10 and 11; science in grades 5, 10, and 11; and social studies in grades 8, 10, and 11. Every TAKS test is directly linked to the Texas Essential Knowledge and Skills (TEKS) curriculum. The TEKS is the state-mandated curriculum for Texas public school students. Essential knowledge and skills taught at each grade build upon the material learned in previous grades. By developing the academic skills specified in the TEKS, students can build a strong foundation for future success.

The Texas Education Agency has developed this study guide to help students strengthen the TEKS-based skills that are taught in class and tested on TAKS. The guide is designed for students to use on their own or for students and families to work through together. Concepts are presented in a variety of ways that will help students review the information and skills they need to be successful on TAKS. Every guide includes explanations, practice questions, detailed answer keys, and student activities. At the end of this book is an evaluation form for you to complete and mail back when you have finished the guide. Your comments will help us improve future versions of this guide.

There are a number of resources available for students and families who would like more information about the TAKS testing program. Information booklets are available for every TAKS subject and grade. Brochures are also available that explain the Student Success Initiative promotion requirements and the new graduation requirements for eleventh-grade students. To obtain copies of these resources or to learn more about the testing program, please contact your school or visit the Texas Education Agency website at www.tea.state.tx.us.

Texas is proud of the progress our students have made as they strive to reach their academic goals. We hope the study guides will help foster student learning, growth, and success in all of the TAKS subject areas.

Sincerely,

Ann Smisko
Associate Commissioner
Curriculum, Assessment, and Technology
Texas Education Agency
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What Is This Book?
This is a study guide to help you strengthen the skills tested on the Grade 10 Texas Assessment of Knowledge and Skills (TAKS). TAKS is a state-developed test administered with no time limit. It is designed to provide an accurate measure of learning in Texas schools.

By acquiring all the skills taught in tenth grade, you will be better prepared to succeed on the Grade 10 TAKS test and during the next school year. This study guide is organized into two sections. This section is about mathematics.

What Are Objectives?
Objectives are goals for the knowledge and skills that you should achieve. The specific goals for instruction in Texas schools were provided by the Texas Essential Knowledge and Skills (TEKS). The objectives for TAKS were developed based on the TEKS.

How Is the Mathematics Section Organized?
The mathematics section of this study guide is divided into the ten objectives tested on TAKS. A statement at the beginning of each objective lists the mathematics skills you need to acquire. The study guide covers a large amount of material. You should not expect to complete it all at once. It may be best to work through one objective at a time.

Each objective is organized into review sections and a practice section. The review sections present examples and explanations of the mathematics skills for each objective. The practice sections feature mathematics problems that are similar to the ones used on the TAKS test.

How Can I Use This Book?
First look at your Confidential Student Report. This is the report the school gave you that shows your TAKS scores. This report will tell you which TAKS subject-area test(s) you passed and which one(s) you did not pass. Use your report to determine which skills need improvement. Once you know which skills need to be improved, you can read through the instructions and examples that support those skills. You may also choose to work through all the sections. Pace yourself as you work through the study guide. Work in short sessions. If you become frustrated, stop and start again later.
What Are the Helpful Features of the Mathematics Section?

- Look for the following features in the margin:
  Ms. Mathematics provides important instructional information for a topic.

Do you see that . . .
  points to a significant sentence in the instruction.

Calculator
  suggests that using a graphing calculator might be helpful.

Memo
  provides page references in this study guide for additional information.

- There are several words in the mathematics section that are important for you to understand. These words are boldfaced in the text and are defined when they are introduced. Locate the boldfaced words and review the definitions.

- Examples are contained inside shaded boxes.

- Each objective has “Try It” problems based on the examples in the review sections.

- A Mathematics Chart for the Grade 10 TAKS test is included on pages 10–11 and also as a tear-out page in the back of the book. This chart includes useful mathematics information. The tear-out Mathematics Chart in the back of the book also provides both a metric and a customary ruler to help solve problems requiring measurement of length.

How Should the “Try It” Problems Be Used?

“Try It” problems are found throughout the review sections of the mathematics study guide. These problems provide an opportunity for you to practice skills that have just been covered in the instruction. Each “Try It” problem features lines for your responses. The answers to the “Try It” problems are found immediately following each problem.

While completing a “Try It” problem, cover up the answer portion with a sheet of paper. Then check the answer.
What Kinds of Practice Questions Are in the Study Guide?

The mathematics study guide contains questions similar to those found on the Grade 10 TAKS test. There are two types of questions in the mathematics section.

- Multiple-Choice Questions: Most of the practice questions are multiple choice with four answer choices. These questions present a mathematics problem using numbers, symbols, words, a table, a diagram, or a combination of these. Read each problem carefully. If there is a table or diagram, study it. You should read each answer choice carefully before choosing the best answer.

- Gridable Questions: Some practice questions use an eight-column answer grid like those used on the Grade 10 TAKS test.

How Do You Use an Answer Grid?

The answer grid contains eight columns, which include three decimal places: tenths, hundredths, and thousandths.

Suppose 5708.61 is the answer to a problem. First write the number in the blank spaces. Be sure to use the correct place value. For example, 5 is in the thousands place, 7 is in the hundreds place, 0 is in the tens place, 8 is in the ones place, 6 is in the tenths place, and 1 is in the hundredths place.

Then fill in the correct bubble under each digit. Notice that if there is a zero in the answer, you need to fill in the bubble for the zero.

The grid shows 5708.61 correctly entered. The zero in the tens place is bubbled in because it is part of the answer. It is not necessary to bubble in the zero in the thousandths place, because this zero will not affect the value of the correct answer.
### Grades 9, 10, and 11 Exit Level Mathematics Chart

<table>
<thead>
<tr>
<th>LENGTH</th>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilometer</td>
<td>1000 meters</td>
<td>1 mile = 1760 yards</td>
</tr>
<tr>
<td>1 meter</td>
<td>100 centimeters</td>
<td>1 mile = 5280 feet</td>
</tr>
<tr>
<td>1 centimeter</td>
<td>10 millimeters</td>
<td>1 yard = 3 feet</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 foot = 12 inches</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CAPACITY AND VOLUME</th>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 liter</td>
<td>1000 milliliters</td>
<td>1 gallon = 4 quarts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 gallon = 128 ounces</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 quart = 2 pints</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 pint = 2 cups</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 cup = 8 ounces</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MASS AND WEIGHT</th>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilogram</td>
<td>1000 grams</td>
<td>1 ton = 2000 pounds</td>
</tr>
<tr>
<td>1 gram</td>
<td>1000 milligrams</td>
<td>1 pound = 16 ounces</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIME</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>365 days</td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td>12 months</td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td>52 weeks</td>
<td></td>
</tr>
<tr>
<td>1 week</td>
<td>7 days</td>
<td></td>
</tr>
<tr>
<td>1 day</td>
<td>24 hours</td>
<td></td>
</tr>
<tr>
<td>1 hour</td>
<td>60 minutes</td>
<td></td>
</tr>
<tr>
<td>1 minute</td>
<td>60 seconds</td>
<td></td>
</tr>
</tbody>
</table>

Metric and customary rulers can be found on the tear-out Mathematics Chart in the back of this book.
Grades 9, 10, and 11 Exit Level Mathematics Chart

<table>
<thead>
<tr>
<th>Area</th>
<th>rectangle</th>
<th>( A = lw ) or ( A = bh )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>triangle</td>
<td>( A = \frac{1}{2}bh ) or ( A = \frac{bh}{2} )</td>
</tr>
<tr>
<td></td>
<td>trapezoid</td>
<td>( A = \frac{1}{2}(b_1 + b_2)h ) or ( A = \frac{(b_1 + b_2)h}{2} )</td>
</tr>
<tr>
<td></td>
<td>circle</td>
<td>( A = \pi r^2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Surface Area</th>
<th>cube</th>
<th>( S = 6s^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cylinder (lateral)</td>
<td>( S = 2\pi rh )</td>
</tr>
<tr>
<td></td>
<td>cylinder (total)</td>
<td>( S = 2\pi rh + 2\pi r^2 ) or ( S = 2\pi r(h + r) )</td>
</tr>
<tr>
<td></td>
<td>cone (lateral)</td>
<td>( S = \pi rl )</td>
</tr>
<tr>
<td></td>
<td>cone (total)</td>
<td>( S = \pi rl + \pi r^2 ) or ( S = \pi r(l + r) )</td>
</tr>
<tr>
<td></td>
<td>sphere</td>
<td>( S = 4\pi r^2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volume</th>
<th>prism or cylinder</th>
<th>( V = Bh^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>pyramid or cone</td>
<td>( V = \frac{1}{3}Bh^* )</td>
</tr>
<tr>
<td></td>
<td>sphere</td>
<td>( V = \frac{1}{3}\pi r^3 )</td>
</tr>
</tbody>
</table>

*B represents the area of the Base of a solid figure.

<table>
<thead>
<tr>
<th>Pi</th>
<th>( \pi )</th>
<th>( \pi = 3.14 ) or ( \pi \approx \frac{22}{7} )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Pythagorean Theorem</th>
<th>( a^2 + b^2 = c^2 )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Distance Formula</th>
<th>( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Slope of a Line</th>
<th>( m = \frac{y_2 - y_1}{x_2 - x_1} )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Midpoint Formula</th>
<th>( M = \left( \frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Quadratic Formula</th>
<th>( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Slope-Intercept Form of an Equation</th>
<th>( y = mx + b )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Point-Slope Form of an Equation</th>
<th>( y - y_1 = m(x - x_1) )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Standard Form of an Equation</th>
<th>( Ax + By = C )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Simple Interest Formula</th>
<th>( I = prt )</th>
</tr>
</thead>
</table>
Objective 1

The student will describe functional relationships in a variety of ways.

For this objective you should be able to recognize that a function represents a dependence of one quantity on another and can be described in a number of ways.

What Is a Function?

A function is a set of ordered pairs \((x, y)\) in which each \(x\)-coordinate is paired with only one \(y\)-coordinate. In a list of ordered pairs belonging to a function, no \(x\)-coordinate is repeated.

When people eat at a restaurant, they often use a tip chart to determine the amount of the tip to leave the server. A tip chart is a good example of a function.

<table>
<thead>
<tr>
<th>Cost of Meal</th>
<th>Recommended Tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.00</td>
<td>$1.00</td>
</tr>
<tr>
<td>$6.00</td>
<td>$1.20</td>
</tr>
<tr>
<td>$7.00</td>
<td>$1.40</td>
</tr>
<tr>
<td>$8.00</td>
<td>$1.60</td>
</tr>
<tr>
<td>$9.00</td>
<td>$1.80</td>
</tr>
<tr>
<td>$10.00</td>
<td>$2.00</td>
</tr>
</tbody>
</table>

On the tip chart above, each meal cost listed has exactly one recommended tip listed. Since the amount of the tip depends on how much the meal costs, the recommended tip is a function of the cost of the meal.

In a functional relationship, for any given input there is a unique output.

If you are given an \(x\)-value belonging to a function, you can find the corresponding \(y\)-value.

If you input $5.00 into the above function, the output will be $1.00.
There are two ways to test a set of ordered pairs to see whether it is a function.

**Examine the list of ordered pairs.**

If a set of ordered pairs is a function, no \( x \)-coordinate in the set is repeated. No \( x \)-coordinate should be listed with two different \( y \)-coordinates.

Is this set of ordered pairs a function?

\[ \{(1, 4), (5, 7), (-1, 7), (10, 12)\} \]

Examine the set of ordered pairs.

- None of the \( x \)-coordinates in the set are repeated.
- Two ordered pairs, \((5, 7)\) and \((-1, 7)\), have the same \( y \)-coordinate but different \( x \)-coordinates. This does not prevent this set of ordered pairs from being a functional relationship.

This set of ordered pairs is a function.

Is this set of ordered pairs a function?

\[ \{(-2, 5), (0, 7), (1, 4), (-2, 6)\} \]

- The number \(-2\) is paired with \(5\); \(0\) is paired with \(7\); \(1\) is paired with \(4\); and \(-2\) is paired with \(6\).
- Two ordered pairs, \((-2, 5)\) and \((-2, 6)\), have the same \( x \)-coordinate. In a functional relationship, no \( x \)-coordinate should repeat.

This set of ordered pairs is not a function.

**Examine a graph of the function.**

Use a vertical line to determine whether two points have the same \( x \)-coordinate. If two points in the function lie on the same vertical line, then they have the same \( x \)-coordinate, and the set of ordered pairs is not a function.
Objective 1

Do the ordered pairs graphed below represent a function?

![Graph image](image-url)

The ordered pairs (2, 5) and (2, 7) lie on a common vertical line. They have the same x-coordinate, 2, but different y-coordinates, 5 and 7.

This graph does not represent a function because two points lie on the same vertical line.

Do you see that . . .

In a function, the y-coordinate is described in terms of the x-coordinate. The value of the y-coordinate depends on the value of the x-coordinate.

Suppose the number of miles you walk is equal to 4 times the number of hours you walk. Which is the dependent quantity in this function?

If you walked for 1 hour, you would have walked 4 miles. If you walked for 3 hours, you would have walked 12 miles.

The distance you walk depends on, or is described in terms of, the number of hours you walk.

In this function, the number of hours you walk is the independent quantity. The distance you walk is the dependent quantity.

An equation that describes this function is \( d = 4h \), where \( d \) represents the number of miles you walk, and \( h \) represents the number of hours. In this equation, \( d \), the distance you walk, depends on \( h \), the number of hours you walk.

- The variable \( h \) is the independent variable.
- The variable \( d \) is the dependent variable.
- The number 4 is a constant, a quantity in an equation that does not change.
Suppose the equation \( c = 0.07m + 0.25 \) describes \( c \), the cost of a
phone call, in terms of \( m \), the number of minutes the phone call
lasts.

In this function, \( c \) is the dependent variable, \( m \) is the independent
variable, and 0.07 and 0.25 are constants.

Jeremy works at an appliance store. He is paid $180.00 a week for
his base salary plus a commission equal to 5% of his total sales.
The equation \( s = 180 + 0.05d \) represents Jeremy’s weekly salary, 
\( s \), in terms of \( d \), his total weekly sales in dollars.

Which variable is the dependent variable in this equation? What
are the constants?
- Jeremy’s weekly salary, \( s \), is the dependent quantity because it
depends on \( d \), his total sales.
- The constants are 180, Jeremy’s base salary, and 0.05,
his commission, because these numbers do not change.

Try It
Cara buys milk for her scout camp each morning. The function
below shows the relationship between \( c \), the total cost of the milk
she buys, and \( n \), the number of quarts she purchases.

\[ c = 1.25n \]

In this functional relationship, which value is the dependent quantity?
The \underline{__________________________} quantity is the number
of quarts of milk Cara purchases.

The \underline{__________________________} quantity is the total
cost of the milk because the cost depends on the number of quarts of
milk Cara buys.

The \textit{independent} quantity is the number of quarts of milk Cara purchases.
The \textit{dependent} quantity is the total cost of the milk because the cost
depends on the number of quarts of milk Cara buys.
**How Can You Represent a Function?**

Functional relationships can be represented in a variety of ways.

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>List</td>
<td>List the ordered pairs.</td>
<td>{(-3, -2), (1, 2), (4, 5), (10.5, 11.5), \ldots}</td>
</tr>
</tbody>
</table>
| Table        | Place the ordered pairs in a table.                       | \[
|              |                                                           | \begin{array}{cc}
|              | \hspace{1em}x & y \\
|              | \hspace{1em}3 & 2 \\
|              | \hspace{1em}1 & 2 \\
|              | \hspace{1em}4 & 5 \\
|              | \hspace{1em}10.5 & 11.5 \\
|              | \ldots & \ldots \\
|              | \end{array}\]                                      |
| Mapping      | Draw a picture that shows how the ordered pairs are formed.| 
| Description  | Use words to describe the functional relationship.        | The \(y\)-values for a set of points are 1 more than the corresponding \(x\)-values. |
| Equation     | Write an equation that describes the \(y\)-coordinate in terms of the \(x\)-coordinate. | \(y = x + 1\)                                                            |
| Function notation | Write a special type of equation that uses \(f(x)\) to represent \(y\). | \(f(x) = x + 1\)                                                       |
| Graph        | Graph the ordered pairs.                                  | ![Graph showing ordered pairs]                                          |
Objective 1

To use function notation to describe a function, give the function a name, typically a letter such as $f$, $g$, or $h$. Then use an algebraic expression to describe the $y$-coordinate of an ordered pair.

Suppose $f(x) = 3x - 1$.

- This function is read as “$f$ of $x$ equals 3 times $x$ minus 1.”
- If you input $x$, the output will be $3x - 1$.
- The $y$-coordinate of the ordered pair is $3x - 1$.

The function described by $f(x) = 3x - 1$ is the same as the function described by $y = 3x - 1$. In this function, an ordered pair looks like this: $(x, 3x - 1)$.

Here are three methods you can use to determine whether two different representations of a function are equivalent.

<table>
<thead>
<tr>
<th>Method</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Match a list or table of ordered pairs to a graph.</td>
<td>● Show that each ordered pair listed matches a point on the graph.</td>
</tr>
<tr>
<td>Match an equation to a graph.</td>
<td>● Determine whether they are both linear or quadratic functions.</td>
</tr>
<tr>
<td></td>
<td>● Find points on the graph and show that their coordinates satisfy the equation.</td>
</tr>
<tr>
<td></td>
<td>● Find points that satisfy the equation and show that they are on the graph.</td>
</tr>
<tr>
<td>Match a verbal description to a graph, an equation, or an expression written in function notation.</td>
<td>● Use the verbal description to find ordered pairs belonging to the function and then show that they satisfy the graph, equation, or function rule.</td>
</tr>
<tr>
<td></td>
<td>● Find points on the graph or ordered pairs satisfying the equation or rule and show that they satisfy the verbal description.</td>
</tr>
</tbody>
</table>
Any equation of the form $y = mx + b$ is a **linear function**. Its graph will be a line.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-11</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Any equation of the form $y = ax^2 + bx + c$ is a **quadratic function**. Its graph will be a parabola.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-5</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>
Which ordered pair, (7, 11) or (−4, −1), belongs to the function in which the y-coordinate is 3 more than the x-coordinate?

Match the ordered pairs to the verbal description.

- For the ordered pair (7, 11), the y-coordinate should be 3 more than 7.
  
  \[ 7 + 3 = 10, \text{ not } 11 \]

  The ordered pair (7, 11) does not belong to this function.

- For the ordered pair (−4, −1), the y-coordinate should be 3 more than −4.
  
  \[ −4 + 3 = −1 \]

  The ordered pair (−4, −1) belongs to this function.

A variety of methods of representing a function are shown below. Which of these examples represents a function that is different from the other functions?

**A. Verbal Description**

The value of \( y \) is double the value of \( x \).

**B. List of Selected Values**

\{ (3, 6), (−2, −4), (1, 2), (0, 0), (−5, −10), \ldots \} 

**C. Table of Selected Values**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1</td>
<td>−2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
</tr>
</tbody>
</table>

**D. Mapping of Selected Values**

![Mapping Diagram]

**E. Graph**

![Graph Diagram]

**F. Equation**

\[ y = 2x \]

Look at the ordered pairs that make up each function.

- In Examples A, B, C, E, and F, each x-coordinate is paired with a y-coordinate that is its double, or two times as great.

- In Example D, for each ordered pair listed, the x-coordinate is paired with a y-coordinate that is two more rather than two times as great. So Example D includes ordered pairs that are not in the other examples.

Only Example D represents a function that is different from the other functions shown.
The table below presents selected values in a functional relationship.

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>1</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

Write an equation that describes this functional relationship.

- Look for a pattern in the ordered pairs that belong to the function.
  
  The y-coordinate appears to be 5 more than the x-coordinate, so \( x + 5 = y \) should represent the pattern.

- Check this equation for each pair.

  For \( x = -2 \)  \( -2 + 5 = 3 \)
  
  For \( x = 1 \)  \( 1 + 5 = 6 \)
  
  For \( x = 5 \)  \( 5 + 5 = 10 \)
  
  For \( x = 7 \)  \( 7 + 5 = 12 \)

The y-coordinate in each ordered pair is 5 more than the x-coordinate.

The rule for this function can be represented by the equation \( x + 5 = y \) or by the equation \( y = x + 5 \).

The table on the right presents selected values in a functional relationship between \( x \) and \( y \).

Using function notation, write a rule that represents the relationship.

- Look for a pattern in the function's ordered pairs.
  
  The y-coordinate appears to be 5 times the x-coordinate plus 2.

- Check this pattern for each pair.

  \[
  5x + 2 = y \\
  (-1, -3) \quad 5 \cdot (-1) + 2 = -3 \\
  (1, 7) \quad 5 \cdot 1 + 2 = 7 \\
  (3, 17) \quad 5 \cdot 3 + 2 = 17 \\
  (8, 42) \quad 5 \cdot 8 + 2 = 42 
  \]

The y-coordinate is equal to 5 times the x-coordinate plus 2.

The rule for this function can be represented by the equation \( y = 5x + 2 \).

Replace \( y \) with \( f(x) \) to express the rule in function notation:

\[ f(x) = 5x + 2. \]
Does the graph below represent the same function as the equation \( y = 3x^2 - 1 \)?

- The function \( y = 3x^2 - 1 \) is a quadratic function because there is an \( x^2 \) term in its equation. Its graph should be a parabola. The above graph is a parabola, so the equation and the graph are the same type of function.

- Show that the equation and the graph represent the same quadratic function with the same set of ordered pairs.

- First check the coordinates of points on the graph to see whether they satisfy the equation. Pick points on the graph whose coordinates are easy to read. For example, you might choose \((0, -1)\), \((1, 2)\), and \((2, 11)\).

Substitute these values into \( y = 3x^2 - 1 \) and determine whether the equation is true.

<table>
<thead>
<tr>
<th>Point ((0, -1))</th>
<th>Point ((1, 2))</th>
<th>Point ((2, 11))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x = 0) and (y = -1)</td>
<td>(x = 1) and (y = 2)</td>
<td>(x = 2) and (y = 11)</td>
</tr>
<tr>
<td>(y = 3x^2 - 1)</td>
<td>(y = 3x^2 - 1)</td>
<td>(y = 3x^2 - 1)</td>
</tr>
<tr>
<td>(-1 \neq 3(0)^2 - 1)</td>
<td>(2 \neq 3(1)^2 - 1)</td>
<td>(11 \neq 3(2)^2 - 1)</td>
</tr>
<tr>
<td>(-1 \neq 3(0) - 1)</td>
<td>(2 \neq 3(1) - 1)</td>
<td>(11 \neq 3(4) - 1)</td>
</tr>
<tr>
<td>(-1 \neq 0 - 1)</td>
<td>(2 \neq 3 - 1)</td>
<td>(11 \neq 12 - 1)</td>
</tr>
<tr>
<td>(-1 = -1)</td>
<td>(2 = 2)</td>
<td>(11 = 11)</td>
</tr>
</tbody>
</table>

The points \((0, -1)\), \((1, 2)\), and \((2, 11)\) are points on the graph, and their coordinates satisfy the equation.

- Next find ordered pairs that satisfy the equation and confirm that the points are on the graph. Pick values that are easy to substitute, like \(x = 1\), \(x = 0\), or \(x = -2\), and find the corresponding values for \(y\).
Substitute these values into \( y = 3x^2 - 1 \) and determine the value for \( y \).

\[
\begin{array}{ccc}
\text{ } & x = 1 & x = 0 & x = -2 \\
y = 3x^2 - 1 & y = 3(1)^2 - 1 & y = 3x^2 - 1 & y = 3(-2)^2 - 1 \\
y = 3(1)^2 - 1 & y = 3(0)^2 - 1 & y = 3(0) - 1 & y = 3(4) - 1 \\
y = 3 - 1 & y = 0 - 1 & y = 12 - 1 & \\
y = 2 & y = -1 & y = 11 & \\
\end{array}
\]

The ordered pairs \((1, 2), (0, -1),\) and \((-2, 11)\) satisfy the equation.

Confirm that these ordered pairs are points on the graph. Yes, all three points are on the graph.

The graph does represent the relationship \( y = 3x^2 - 1 \).

**Try It**

The equation \( y = x^2 - 1 \) represents a functional relationship. Which graph represents this function?

Determine whether the equation is a linear or quadratic function.

The equation is a ________________ function because it contains the term \( x^2 \), which has an exponent of 2.

Its graph must be a ________________.
Answer choices ______ and ______ cannot be the graph of this function because they are ________________.

Determine which parabola is the correct graph.

See whether the point (0, 1) in answer choice B satisfies the equation \( y = x^2 - 1 \).

When \( x = 0 \) and \( y = \) ______, is the equation \( y = x^2 - 1 \) true?

Does ______ = ______ - ______?

No, ______ ≠ ______.

Answer choice B is ________________.

See whether the point (0, -1) in answer choice C satisfies the equation \( y = x^2 - 1 \).

When \( x = \) ______ and \( y = \) ______, is the equation \( y = x^2 - 1 \) true?

Does ______ = ______ - ______?

Yes, ______ = ______.

Answer choice C is ________________.

The equation is a **quadratic** function because it contains the term \( x^2 \), which has an exponent of 2. Its graph must be a **parabola**. Answer choices A and D cannot be the graph of this function because they are **lines**. When \( x = 0 \) and \( y = 1 \), is the equation \( y = x^2 - 1 \) true? Does \( 1 = 0^2 - 1 \)? No, \( 1 ≠ -1 \). Answer choice B is **not correct**. When \( x = 0 \) and \( y = -1 \), is the equation \( y = x^2 - 1 \) true? Does \( -1 = 0^2 - 1 \)? Yes, \(-1 = -1 \). Answer choice C is **correct**.
Objective 1

How Can You Draw Conclusions from a Functional Relationship?

Use these guidelines when interpreting functional relationships in a real-life problem.

- Understand the problem.
- Identify the quantities involved and any relationships between them.
- Determine what the variables in the problem represent.
- For graphs: Determine what quantity each axis on the graph represents. Look at the scale that is used on each axis.
- For tables: Determine what quantity each column in the table represents.
- Look for trends in the data. Look for maximum and minimum values in graphs.
- Look for any unusual data. For example, does a graph start at a nonzero value? Is one of the problem's variables negative at any point?
- Match the data to the equations or formulas in the problem.

The graph below shows the daily high temperatures in degrees Fahrenheit over a two-week period in August. During which three-day period did the temperature decrease by the greatest number of degrees?

The temperature decreased from August 14 to August 16 and then again from August 17 to August 21. To determine which three-day period it decreased by the greatest number of degrees, you need to find the coordinates of these points and calculate the total drop in the daily high temperature.

You could build a table to organize your work.
Objective 1

The high temperature decreased by 18°F from August 19 to August 21. This was the greatest decrease in temperature for any three-day period on the graph.

<table>
<thead>
<tr>
<th>3-Day Period</th>
<th>Day 1 High Temperature</th>
<th>Day 2 High Temperature</th>
<th>Day 3 High Temperature</th>
<th>Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aug. 14, 15, 16</td>
<td>105°F</td>
<td>103°F</td>
<td>100°F</td>
<td>5°F</td>
</tr>
<tr>
<td>Aug. 17, 18, 19</td>
<td>102°F</td>
<td>100°F</td>
<td>98°F</td>
<td>4°F</td>
</tr>
<tr>
<td>Aug. 18, 19, 20</td>
<td>100°F</td>
<td>98°F</td>
<td>85°F</td>
<td>15°F</td>
</tr>
<tr>
<td>Aug. 19, 20, 21</td>
<td>98°F</td>
<td>85°F</td>
<td>80°F</td>
<td>18°F</td>
</tr>
</tbody>
</table>

The formula for converting degrees Celsius, C, to degrees Fahrenheit, F, is $F = \frac{9}{5}C + 32$.

The points increase at a rate of 9°F for every 5°C, and the slope of the graph should be $\frac{9}{5}$, the coefficient of $C$.

Two points on the graph are (0, 32) and (100, 212). When the coordinates are substituted into the equation $F = \frac{9}{5}C + 32$, the equation is true.

\[
\begin{align*}
F &= \frac{9}{5}C + 32 \\
212 &= \frac{9}{5}(100) + 32 \\
212 &= 180 + 32 \\
212 &= 212
\end{align*}
\]

Yes, the graph accurately represents this function.

Now practice what you’ve learned.
Objective 1

Question 1
Rhonda works at a grocery store after school. She is paid $5.50 per hour. Her weekly salary, \( s \), is described by the function \( s = 5.5h \), where \( h \) is the number of hours she works in a week. What is the dependent quantity in this functional relationship?

A  The number of hours she works in a week  
B  The number of dollars she is paid per hour  
C  The total salary for a week  
D  The number of days she works in a week  

Answer Key: page 224

Question 2
The number of pretzels, \( p \), that can be packaged in a box with a volume of \( V \) cubic units is given by the equation \( p = 45V + 10 \). In this relationship, which is the dependent variable?

A  10  
B  45  
C  \( p \)  
D  \( V \)  

Answer Key: page 224

Question 3
Which of the following tables does not represent a function?

\[
\begin{array}{c|c|}
 x & y \\
 \hline
 1 & 2 \\
 4 & 1 \\
 -1 & 2 \\
 -4 & 1 \\
\end{array}
\]

A  

\[
\begin{array}{c|c|}
 x & y \\
 \hline
 1 & 1 \\
 -2 & 4 \\
 3 & 9 \\
 1 & 16 \\
\end{array}
\]

B  

\[
\begin{array}{c|c|}
 x & y \\
 \hline
 -1 & -1 \\
 3 & 3 \\
 2 & 2 \\
 5 & 5 \\
\end{array}
\]

C  

\[
\begin{array}{c|c|}
 x & y \\
 \hline
 2 & 0 \\
 1 & 1 \\
 3 & 5 \\
 4 & 0 \\
\end{array}
\]

D  

Answer Key: page 224

Question 4
The table shows the independent and dependent values in a functional relationship. Which function best represents this relationship?

<table>
<thead>
<tr>
<th>Independent</th>
<th>Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
</tr>
</tbody>
</table>

A  \( f(x) = 2x^2 - 2x + 1 \)  
B  \( f(x) = 2x^2 + 2x + 1 \)  
C  \( f(x) = x^2 + 1 \)  
D  \( f(x) = 8x + 9 \)  

Answer Key: page 224
**Question 5**

Jane started a weekend pet-care business. She bought the necessary supplies for $210. Jane charges $25 per weekend for each pet she cares for. Which function best represents her net profit in terms of \(x\), the number of pets she cares for?

- **A** \(f(x) = x + 25 - 210\)
- **B** \(f(x) = 210 - 25x\)
- **C** \(f(x) = 210x - 25\)
- **D** \(f(x) = 25x - 210\)

**Answer Key: page 224**

**Question 6**

Jeb's stereo is playing at a volume of 75 decibels. If the decibel level reaches 120 decibels, the neighbors will complain. Which inequality models \(q\), the number of decibels Jeb can increase the volume before the neighbors complain?

- **A** \(75 + q < 120\)
- **B** \(75 - q > 120\)
- **C** \(75 + q > 120\)
- **D** \(75 - q < 120\)

**Answer Key: page 224**

**Question 7**

Which table best represents the function \(f(x) = \frac{2}{3}x - 3\)?

- **A**
<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>10</td>
<td>-1</td>
</tr>
<tr>
<td>15</td>
<td>9</td>
</tr>
</tbody>
</table>

- **B**
<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>-7</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

- **C**
<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-5</td>
</tr>
<tr>
<td>20</td>
<td>-1</td>
</tr>
<tr>
<td>30</td>
<td>9</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
</tr>
</tbody>
</table>

- **D**
<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-7</td>
</tr>
<tr>
<td>30</td>
<td>9</td>
</tr>
<tr>
<td>60</td>
<td>-21</td>
</tr>
<tr>
<td>80</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Answer Key: page 224**
**Objective 1**

**Question 8**

Which graph best represents the function \( f(x) = x^2 - 9 \)?

A

B

C

D

Answer Key: page 225
**Question 9**

In chemistry lab Sonya places an empty beaker with a mass of 35 grams on a scale. She adds 5-gram measures of water to the beaker until the total mass of the beaker and water is 85 grams. Each time Sonya adds 5 grams of water, she records the mass. Which graph best illustrates the combined mass of the beaker and water as the amount of water in the beaker increases?

- **A**
- **B**
- **C**
- **D**

*Answer Key:* page 225
Question 10
Jesse sells peanut butter cookies and chocolate chip cookies to raise money for his club. On average, he sells about twice as many chocolate chip cookies as he does peanut butter cookies. Which graph best models this relationship?

A

B

C

D

Answer Key: page 225
**Question 11**

Members of the junior class are raising money for their class trip by selling artwork by local artists. They bought the merchandise from the artists for a total price of $1025. The graph shows their cumulative total income for the two weeks of the sale. On what day of the sale did they first make a profit?

A  Day 5  
B  Day 4  
C  Day 0  
D  Day 14

**Question 12**

Potassium nitrate dissolves in water to form a solution. The graph below shows the solubility of potassium nitrate in water as a function of the temperature of the water.

According to this graph, about how many grams of potassium nitrate will dissolve in 100 cm³ of water at 42°C?

A  58 grams  
B  82 grams  
C  77 grams  
D  68 grams
Objective 1

Question 13
A home-builders group recently published a study comparing the cost of building homes from 1,000 to 3,000 square feet in area in four different communities. The study found that the formulas below predicted the approximate cost, $c$, of building a new home in each of these communities in terms of $f$, the area of the home in square feet.

<table>
<thead>
<tr>
<th>Community</th>
<th>Cost of Building a New Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>$c = 15,000 + 80f$</td>
</tr>
<tr>
<td>S</td>
<td>$c = 25,000 + 75f$</td>
</tr>
<tr>
<td>T</td>
<td>$c = 60,000 + 50f$</td>
</tr>
<tr>
<td>V</td>
<td>$c = 40,000 + 65f$</td>
</tr>
</tbody>
</table>

Based on these formulas, in which community would it cost the least to build a home with an area of 1,450 square feet?

A  T  
B  S  
C  R  
D  V  

Answer Key: page 226
Objective 2

The student will demonstrate an understanding of the properties and attributes of functions.

For this objective you should be able to

- use the properties and attributes of functions;
- use algebra to express generalizations and use symbols to represent situations; and
- manipulate symbols to solve problems and use algebraic skills to simplify algebraic expressions and solve equations and inequalities in problem situations.

What Are Parent Functions?
The simplest linear function, $y = x$, is the *linear parent function*.

- If the graph of any function is a line, then its parent function is $y = x$.
- A linear equation never has variables raised to a power other than 1.
- If an equation is linear, then its parent function is $y = x$. 
What is the parent function of this graph?

Since the graph of $y = x - 5$ is a line, its parent function is the linear parent function, $y = x$. 

Parent function

$y = x$
What is the parent function of the equation \( y = -\frac{1}{2}x - 1 \)?
Since the equation \( y = -\frac{1}{2}x - 1 \) is a linear equation, its graph is a line. Its parent function is the linear parent function, \( y = x \).

The simplest quadratic function, \( y = x^2 \), is the quadratic parent function.

- If the graph of any function is a parabola, then its parent function is \( y = x^2 \).
- If an equation can be written in the form \( y = ax^2 + bx + c \), then it is quadratic.
- If an equation can be written in this form, then its parent function is \( y = x^2 \).
What is the parent function of this graph?

Since the graph of \( y = x^2 + 3 \) is a parabola, its parent function is the quadratic parent function, \( y = x^2 \).
What is the parent function of \( y = -x^2 \)?

The equation \( y = -x^2 \) is a quadratic equation; therefore, its parent function is the quadratic parent function, \( y = x^2 \).

**What Are the Domain and Range of a Function?**

A function is a set of ordered pairs of numbers \((x, y)\) such that no \( x \)-values are repeated. The domain and range of a function are sets that describe those ordered pairs.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Example ((0,1), (2, 6), (3, 5))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain</td>
<td>All the ( x )-coordinates in the function’s ordered pairs</td>
</tr>
<tr>
<td>Range</td>
<td>All the ( y )-coordinates in the function’s ordered pairs</td>
</tr>
</tbody>
</table>

- The **domain** is the set of all the values of the independent variable, the \( x \)-coordinate.
- The **range** is the set of all the values of the dependent variable, the \( y \)-coordinate.

Identify the domain and range of the function below.

\[ \{(3, 9), (5, 39), (9, 23), (6, 14)\} \]

The domain is the set of \( x \)-coordinates in the ordered pairs: \( \{(3, 9), (5, 39), (9, 23), (6, 14)\} \). The domain is \( \{3, 5, 6, 9\} \).

The range is the set of \( y \)-coordinates in the ordered pairs: \( \{(3, 9), (5, 39), (9, 23), (6, 14)\} \). The range is \( \{9, 14, 23, 39\} \).
The domain and range of algebraic functions are usually assumed to be the set of all real numbers. In some cases, however, the domain or range of a function may be a subset of the real numbers because certain numbers would not make sense in a real-life problem situation.

Try It
What are the domain and range of the function below?
\{(4, 9), (-5, 16), (6, 25), (7, -36)\}

The domain of a function is the set of all \(x\)-coordinates.
The domain of this function is \{4, -5, 6, 7\}.
The range of a function is the set of all \(y\)-coordinates.
The range of this function is \{9, 16, 25\}.

The domain of a function is the set of all \(x\)-coordinates. The domain of this function is \{-5, 4, 6, 7\}. The range of a function is the set of all \(y\)-coordinates. The range of this function is \{-36, 9, 16, 25\}.

Consider the function \(l = 4h\), in which \(l\) equals the number of legs on \(h\) horses. Are there any values that would not be reasonable to include in the domain or range of this function?

- The domain of this function is the set of values you may choose for \(h\), the independent variable. Would it be reasonable to let \(h = 1.2\)? No. The variable \(h\) represents a number of horses; it must be a nonnegative integer. The domain is the set of nonnegative integers, \{0, 1, 2, 3, \ldots\}.

It would not be reasonable to include any other numbers in the domain.

- The range of this function is the set of values you will obtain for the dependent variable, \(l\), the number of legs for a group of \(h\) horses. Is it possible to get 6 as a value for \(l\)? Could a group of horses normally have 6 legs? No, 6 is not a reasonable value for the range of this function. Since 1 horse has 4 legs, 2 horses have 8 legs, and so on, the range of this function is the set of multiples of 4, or \{0, 4, 8, 12, \ldots\}.

It would not be reasonable to include any other numbers in the range.
The total volume in sales generated by a television commercial is a function of the number of people who watch the commercial. Wilson Electronics executives estimate that \( s \), the dollars they will generate in sales for the number of people, \( n \), who watch their commercial, is described by the function \( s = 0.0035n \).

What set of numbers would be an appropriate domain for this function: the integers, the real numbers, the rational numbers, or the whole numbers?

The domain of this function is the number of people who watch the commercial. The number of people cannot be a fraction or a negative number. Of the choices given, the only set of numbers that does not contain any fractions or negative numbers is the whole numbers.

The set of whole numbers would be an appropriate domain for this function.

The graph of a function also can tell you about its domain and range.

- The domain of a function is the set of all the \( x \)-coordinates in the function's graph.
- The range of a function is the set of all the \( y \)-coordinates in the function's graph.
What inequalities best describe the domain and range of the function represented in this graph?

- The domain of this function is the set of \( x \)-values in the graph.
- The range of this function is the set of \( y \)-values in the graph.

**Domain**

**Range**

\[ 0 \leq y \leq 16 \]

\[ 1 \leq x < 9 \]
In a chemistry experiment two chemicals are poured into a beaker to react. The temperature of the solution is taken every minute for 10 minutes beginning at 1:05 P.M. The graph below shows the data from this experiment.

What is a reasonable range for this function?

The range of the function is the set of all the y-coordinates in the function. The y-coordinates represent the temperature of the mixed chemicals. The temperature during the 10-minute interval begins at 10°C, and it ends at 50°C.

A reasonable range for this function is $10 \leq y \leq 50$. 
The number of pounds of potato salad, \( p \), that will be needed for a company picnic is given by the function \( p = 0.25n + 4 \), in which \( n \) equals the number of people who will attend the picnic. Each employee in the company can attend the picnic, and each can bring 3 guests. A total of 12 employees and guests have already signed up to attend the picnic. If the company employs a total of 40 people, what is a reasonable range for this function?

The range of the function is the set of all the possible values for the \( \underline{\text{dependent variable}} \) in the function, the amount of potato salad to be purchased.

To determine the range of the function, first determine the minimum and \( \underline{\text{maximum number of people}} \) who will attend the picnic.

The minimum number of people is ________, since that many people have already signed up.

The company has ________ employees. If every employee attends and brings 3 guests, then the maximum number of people is ________ because \( 40 \cdot \underline{\text{number}} = \underline{\text{number}} \).

Use the function \( p = \underline{\text{function}} \) to find the number of pounds of potato salad that will be needed.

If 12 people attend, then \( p = \underline{\text{number}} \cdot \underline{\text{number}} + 4 \); ________ pounds of potato salad will be needed.

If 160 people attend, then \( p = \underline{\text{number}} \cdot \underline{\text{number}} + 4 \); ________ pounds of potato salad will be needed.

The number of pounds of potato salad that will be needed is between ________ pounds and ________ pounds.

A reasonable range for this function is ________ \( \leq p \leq \underline{\text{number}} \).

The range of the function is the set of all the possible values for the \( \underline{\text{dependent variable}} \) in the function, the amount of potato salad to be purchased. To determine the range of the function, first determine the minimum and \( \underline{\text{maximum number of people}} \) who will attend the picnic. The minimum number of people is 12, since that many people have already signed up. The company has 40 employees. If every employee attends and brings 3 guests, then the maximum number of people is 160 because \( 40 \cdot 3 = 160 \). Use the function \( p = 0.25n + 4 \) to find the number of pounds of potato salad that will be needed. If 12 people attend, then \( p = 0.25 \cdot 12 + 4 \); 7 pounds of potato salad will be needed. If 160 people attend, then \( p = 0.25 \cdot 160 + 4 \); 44 pounds of potato salad will be needed. The number of pounds of potato salad that will be needed is between 7 pounds and 44 pounds. A reasonable range for this function is \( 7 \leq p \leq 44 \).
How Can You Interpret a Problem from a Graph?

To interpret a problem situation described in terms of a graph, follow these guidelines.

- Identify the quantities that are being compared.
- Understand what relationship the graph is describing.
- Look at the scales used on the axes of the graph.
- Identify the domain or range of the function graphed.
- Look for patterns in the data—increases, decreases, or data that remain constant.

Joe walked at a constant speed. The graph represents his distance from his starting point in terms of the time he walked.

Give a reasonable description of the route Joe walked.

The y-axis represents Joe’s distance from his starting point. The x-axis represents his walking time. Did Joe get farther and farther away from his starting point as time went by? No, not for the entire time graphed.

At first Joe’s distance from his starting point was 0. As time went by, this distance increased, but then it returned to 0. He ended his walk where he began it.

A reasonable description of the route Joe walked is that he started at a certain point, walked for a while in one direction, and then turned around and returned to his starting point. The graph represents such a route.
Try It
Terri placed a pot of water on the stove to boil. The temperature of the water in terms of the time it was on the stove is represented by the graph.

What is a reasonable interpretation of the graph?
At first the water temperature _____________________.
Then the water temperature remained ________________ for a while.
Finally the water temperature ________________ slowly.
A reasonable interpretation of the graph is that the water temperature ________________ until the water boiled. Then it remained at a ________________ temperature until Terri turned the stove off. Finally it ________________ slowly to room temperature, where it remained constant.

At first the water temperature increased. Then the water temperature remained constant for a while. Finally the water temperature decreased slowly. A reasonable interpretation of the graph is that the water temperature increased until the water boiled. Then it remained at a constant temperature until Terri turned the stove off. Finally it cooled slowly to room temperature, where it remained constant.
What Is a Correlation in a Scatterplot?

One way to represent a set of related data is to graph the data using a scatterplot. In a scatterplot each pair of corresponding values in the data set is represented by a point on a graph.

To make predictions using a scatterplot, look for a correlation, or pattern, in the data. The pattern may not be true for every point, but look for the overall pattern the data seem to best fit.
As you move from left to right on the graph, if the data points ...

<table>
<thead>
<tr>
<th>As shown in this scatterplot ...</th>
<th>they show this type of correlation:</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Scatterplot" /> move up</td>
<td><img src="image2.png" alt="Scatterplot" /> positive correlation</td>
</tr>
<tr>
<td><img src="image3.png" alt="Scatterplot" /> move down</td>
<td><img src="image4.png" alt="Scatterplot" /> negative correlation</td>
</tr>
<tr>
<td><img src="image5.png" alt="Scatterplot" /> show no pattern</td>
<td><img src="image6.png" alt="Scatterplot" /> no correlation</td>
</tr>
<tr>
<td><img src="image7.png" alt="Scatterplot" /> show a horizontal or vertical pattern</td>
<td><img src="image8.png" alt="Scatterplot" /> undefined correlation</td>
</tr>
</tbody>
</table>
In a survey of property values, \( p \), the price of an acre of land, was compared to \( d \), the distance of the land from the center of town. The data were graphed in a scatterplot. Describe the correlation between the cost of an acre of land and the land's distance from the center of town.

Look for a pattern in the graph. The general tendency is for the price of an acre of land to decrease as the land's distance from the center of town increases. The price of an acre of land has a negative correlation to the land's distance from the center of town.

Try It

Raúl is a sport fisherman. He weighs each fish he catches, and he measures its length. He graphed his data in a scatterplot.

Describe the correlation between the lengths and weights of the fish Raúl caught.

As the lengths of the fish _____________, their weights generally ________________.

This is a ________________ correlation.

As the lengths of the fish increase, their weights generally increase. This is a positive correlation.
Objective 2

How Do You Use Symbols to Represent Unknown Quantities?

Represent unknown quantities with variables, or letters such as $x$ or $y$. Use variables in expressions, equations, or inequalities.

Jacob, Barbara, and Felix all have part-time jobs after school. Jacob earns $5 more per week than Felix, and Barbara earns twice as much per week as Jacob. The combined earnings of these three students are $115 per week. Write an equation that can be used to find how much each student earns per week.

- Represent the number of dollars each student earns per week.
  - $x$ = Felix's earnings
  - $x + 5$ = Jacob's earnings ($5$ more than Felix's earnings)
  - $2(x + 5)$ = Barbara's earnings (twice Jacob's earnings)

- Represent the sum of their earnings.
  - $x + (x + 5) + 2(x + 5)$

- Write an equation setting this sum equal to $115$.
  - $x + (x + 5) + 2(x + 5) = 115$

Quattro plans to paint his living room walls. The room is 3 feet longer than it is wide. The walls are 8 feet high. If a can of paint covers approximately 200 ft$^2$, what expression can be used to represent the minimum number of cans of paint Quattro will need?

- Represent the area of each wall.
  - Smaller Walls
    - $x$ = width of the room
    - 8 = height of the room
    - 8$x$ = area
  - Larger Walls
    - $x + 3$ = length of the room
    - 8 = height of the room
    - 8$(x + 3)$ = area

- Represent the sum of the areas of the four walls.
  - 2(smaller wall) + 2(larger wall)
    - 2(8$x$) + 2[8(x + 3)]
    - 16$x$ + 16(x + 3)
    - 16$x$ + 16$x$ + 48
    - 32$x$ + 48

The total area of the four walls is represented by the expression $32$x + 48$. Since the paint in each can covers 200 ft$^2$, divide the area by 200 to represent the number of cans of paint needed: $\frac{32x + 48}{200}$. 
How Do You Represent Patterns in Data Algebraically?

To represent patterns in data algebraically, follow these guidelines.

- Identify what quantities the data represent.
- Identify the relationships between those quantities.
- Look for patterns in the data.
- Use symbols to translate the patterns into an algebraic expression or equation.

Ronald and Jaime make weekly deposits into their savings accounts. The table below shows the opening balances and the balances for each account after the first four weekly deposits.

<table>
<thead>
<tr>
<th>Name</th>
<th>Opening Balance</th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ronald</td>
<td>$100</td>
<td>$124</td>
<td>$148</td>
<td>$172</td>
<td>$196</td>
</tr>
<tr>
<td>Jaime</td>
<td>$180</td>
<td>$198</td>
<td>$216</td>
<td>$234</td>
<td>$252</td>
</tr>
</tbody>
</table>

If the pattern continues, what expression can be used to represent the balance in Jaime's account after \( n \) weeks?

Jaime's opening balance is $180. His balance increases by $18 each week. His balance in \( n \) weeks can be represented by the expression \( 18n + 180 \). See whether this expression works for all the known values.

<table>
<thead>
<tr>
<th>Week</th>
<th>( 18n + 180 )</th>
<th>Balance (dollars)</th>
<th>Yes/No</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 18 \cdot 1 + 180 = 198 )</td>
<td>198</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>( 18 \cdot 2 + 180 = 216 )</td>
<td>216</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>( 18 \cdot 3 + 180 = 234 )</td>
<td>234</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>( 18 \cdot 4 + 180 = 252 )</td>
<td>252</td>
<td>Yes</td>
</tr>
</tbody>
</table>

If the pattern continues, the balance in Jaime's account after \( n \) weeks can be represented by the expression \( 18n + 180 \).
The table below represents a functional relationship between $x$, the lengths of the bases of a series of triangles, and $y$, their heights. If the pattern continues, what expression can be used to express their heights in terms of their bases?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>

The differences between the $y$-values are not constant. For example, $5 - 2 = 3$, but $10 - 5 = 5$. This is not a linear relationship.

To determine whether the relationship is quadratic, compare the $y$-values to the square of the $x$-values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>17</td>
</tr>
</tbody>
</table>

The $y$-values appear to be 1 more than the squares of the $x$-values. See whether all the coordinates satisfy the rule $y = x^2 + 1$.

Substitute the values of $x$ and $y$ from the table into the equation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = x^2 + 1$</th>
<th>$y$</th>
<th>Yes/No</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2 = (1)^2 + 1$</td>
<td>2</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>$5 = (2)^2 + 1$</td>
<td>5</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>$10 = (3)^2 + 1$</td>
<td>10</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>$17 = (4)^2 + 1$</td>
<td>17</td>
<td>Yes</td>
</tr>
</tbody>
</table>

All the ordered pairs satisfy the equation $y = x^2 + 1$. Since every pair in the table satisfies the equation, then $y = x^2 + 1$ expresses the triangles' heights in terms of their bases.
How Do You Simplify Algebraic Expressions?

You can use the commutative, associative, and distributive properties to simplify algebraic expressions. Use these properties to remove parentheses and combine like terms.

<table>
<thead>
<tr>
<th>Name</th>
<th>Property (If ( a, b, ) and ( c ) are any three real numbers, then...)</th>
<th>Examples</th>
</tr>
</thead>
</table>
| Commutative Property  | \( a + b = b + a \)
                        | or \( ab = ba \)                                                           | \( 4 + 2x = 2x + 4 \)
                        |                                                                  | \( 5y^2x = 5xy^2 \)         |
| Associative Property  | \( a + (b + c) = (a + b) + c \)
                        | or \( a(bc) = (ab)c \)                                                      | \( 5 + y + 2y = 5 + (y + 2y) \)
                        |                                                                  | \( = 5 + 3y \)              |
                        |                                                                    | \( 5(3m) = (5 \cdot 3)m \)                                                   | \( = 15m \)                |
| Distributive Property | \( a(b + c) = ab + ac \)                                                      | \( 7(2x + 3) = 7(2x) + 7(3) \)
                        |                                                                     | \( = 14x + 21 \)             |

- Use the commutative property to change the order of the terms in addition and multiplication.
- Use the associative property to change the groupings in addition or multiplication.
- Use the distributive property to remove the parentheses by multiplying the term outside the parentheses by each term inside the parentheses.

Like terms are terms in an algebraic expression that use the same variable raised to the same power.

For example, in the expression \( 6t^3 - 2t^3 \), \( 6t^3 \) and \( 2t^3 \) are like terms because they are both expressed in terms of the same variable, \( t \), raised to the same power, 3.

Like terms in an algebraic expression can be combined. \( 6t^3 - 2t^3 = 4t^3 \)
If the length of a rectangle is represented by \( l \) and its width is 3 units less than its length, does the expression \( l^2 - 3l \) represent its area?

- Let \( l \) represent the length of the rectangle.
- Let \( l - 3 \) represent the width (3 units less than the length).
- Then \( l(l - 3) \) represents the area of the rectangle, \( A = lw \).

Simplify the expression \( l(l - 3) \).

\[
l(l - 3) = l \cdot l - l \cdot 3 = l^2 - 3l
\]

Yes, the expression \( l^2 - 3l \) represents the area of the rectangle.
The lengths of the three sides of a triangle are represented by the expressions $2m + 5$, $m - 1$, and $7m + 3$. Write an expression in terms of $m$ that can be used to represent the perimeter of the triangle.

- The perimeter of a triangle is the sum of the lengths of its three sides. The perimeter of the triangle can be represented by the expression $2m + 5 + m - 1 + 7m + 3$.
- Simplify this expression.

$$2m + 5 + m - 1 + 7m + 3 = (2m + 1m + 7m) + (5 - 1 + 3)$$
$$= 10m + 7$$

The perimeter of the triangle can be represented by the expression $10m + 7$.

Simplify the expression $(4b^2 + 6) - (2b^2 + 3)$.

When parentheses are preceded by a negative sign, it means the quantity in parentheses is multiplied by $-1$.

$$(4b^2 + 6) - (2b^2 + 3) = (4b^2 + 6) + -1(2b^2 + 3)$$
$$= 4b^2 + 6 - 2b^2 - 3$$
$$= (4b^2 - 2b^2) + (6 - 3)$$
$$= 2b^2 + 3$$

When simplified, the expression $(4b^2 + 6) - (2b^2 + 3) = 2b^2 + 3$. 
Mr. and Mrs. Seymour have a dog pen in their backyard, as shown by the shaded figure in the diagram below.

Write an expression that can be used to represent the area of the yard that does not include the dog pen.

- The area of the entire yard, a rectangle, is equal to its width times its length.
  
  The area of the rectangle is $x(2x + 6)$. Simplify.
  
  $x(2x + 6) = 2x^2 + 6x$

- The area of the dog pen is equal to its length times its width. The area of the smaller rectangle is $(x - 12)(x - 16)$. Simplify by using the FOIL method.
  
  $(x - 12)(x - 16) = (x \cdot x) + (x \cdot -16) + (-12 \cdot x) + (-12 \cdot -16)
  
  = x^2 + (-16x - 12x) + 192
  
  = x^2 - 28x + 192$

- Subtract to find the area of the yard not including the dog pen.
  
  Area of yard - Area of pen
  
  $(2x^2 + 6x) - (x^2 - 28x + 192)$

- Simplify by multiplying the quantities in the second parentheses by $-1$.
  
  $(2x^2 + 6x) + -1(x^2 - 28x + 192) = 2x^2 + 6x - x^2 + 28x - 192
  
  = (2x^2 - x^2) + (6x + 28x) - 192
  
  = x^2 + 34x - 192$

The expression $x^2 + 34x - 192$ represents the area of the yard that does not include the dog pen.
How Do You Solve Algebraic Equations?
To solve algebraic equations, follow these guidelines.

- Simplify any expressions in the equation.
- Add or subtract on both sides of the equation to get variable terms on one side and constant terms on the other.
- Simplify again if necessary.
- Multiply or divide to obtain an equation that has the variable isolated with a coefficient of 1.

Solve the equation $3(y + 2) = -9$.

\[
3(y + 2) = -9 \\
3y + 3(2) = -9 \\
3y + 6 = -9 \\
\underline{-6} = \underline{-6} \\
3y = -15 \\
\frac{3y}{3} = \frac{-15}{3} \\
y = -5
\]

In this equation, $y = -5$.

Solve the equation $2x - 5 = x + 4$.

\[
2x - 5 = x + 4 \\
-1x = -1x \\
x - 5 = 4 \\
+ 5 = + 5 \\
x = 9
\]

In this equation, $x = 9$. 

In the term $6x$, 6 is called the coefficient of $x$. 
In the figure below, the perimeter of the rectangle is 20 units greater than the perimeter of each triangle. Find the length of the diagonal of the rectangle.

- **Represent the perimeter of the rectangle.**
  
  \[ P = 2l + 2w \]
  
  \[ = 2(3x - 5) + 2(2x) \]

- **Simplify this expression.**
  
  \[ 2(3x - 5) + 2(2x) = 2(3x) - 2(5) + (2 \cdot 2)x \]
  
  \[ = 6x - 10 + 4x \]
  
  \[ = (6x + 4x) - 10 \]
  
  \[ = 10x - 10 \]

- **Represent the perimeter of a triangle.**
  
  \[ P = s_1 + s_2 + s_3 \]
  
  \[ = (3x - 5) + (2x) + (4x - 10) \]

- **Simplify this expression.**
  
  \[ (3x - 5) + (2x) + (4x - 10) = 3x - 5 + 2x + 4x - 10 \]
  
  \[ = (3x + 2x + 4x) + (-5 - 10) \]
  
  \[ = 9x - 15 \]

- **Write an equation.**
  
  Perimeter of rectangle = Perimeter of triangle + 20
  
  \[ 10x - 10 = 9x - 15 + 20 \]
  
  \[ 10x - 10 = 9x + 5 \]
  
  \[ -9x = -9x \]
  
  \[ x - 10 = 5 \]
  
  \[ +10 = +10 \]
  
  \[ x = 15 \]
Use this value for $x$ to find the length of the diagonal of the rectangle. The diagonal is represented by the expression $4x - 10$. Replace $x$ with 15.

$$4x - 10 = 4(15) - 10$$
$$= 60 - 10$$
$$= 50$$

The diagonal of the rectangle is 50 units long.
Try It
A manufacturer builds cubical containers. The equation that models
\( c \), the cost of building a container, is
\[
3.6s + 3.20
\]
in which \( s \) represents the length of each side of the cubical container in feet.

Find the length of each side of a cubical container that costs $35.60
to manufacture.

Substitute \( \underline{35.60} \) for \( c \) in the equation that models the cost of
building a container.

Solve the equation for \( \underline{s} \), the length of a side.

\[
\begin{align*}
35.60 &= 3.6s + 3.20 \\
-3.20 &= -3.20 \\
\hline
32.40 &= 3.6s \\
\underline{\hline}
\end{align*}
\]

\( \underline{s} = \underline{\frac{32.40}{3.6}} \)

Each side of the cubical container is \( \underline{9} \) feet long.

Substitute \( 35.60 \) for \( c \) in the equation that models the cost of building a
container. Solve the equation for \( s \), the length of a side.

\[
\begin{align*}
35.60 &= 3.6s + 3.20 \\
-3.20 &= -3.20 \\
\hline
32.40 &= 3.6s \\
\underline{\hline}
\end{align*}
\]

\( \underline{s} = \underline{\frac{32.40}{3.6}} \)

Each side of the cubical container is \( \underline{9} \) feet long.

Now practice what you've learned.
Question 14
Which function is the parent function of the graph below?

A $y = 2x$
B $y = x^2$
C $x = y^2$
D $y = x$

Answer Key: page 226

Question 16
Which of the following situations is best represented by the graph below?

A The speed of a roller coaster as it goes from a high point on its track to a low point on the track and then back up to another high point
B The price of a stock that drops to half of its original value and then goes back to its original value
C The speed of a bullet that is fired straight up and then falls back to the ground
D The speed of a race car that starts from the starting line, races several laps, and then makes a refueling stop

Answer Key: page 226

Question 15
Marcy deposits $550 in a savings account at 3% simple annual interest. The value of this account, $v$, is given by the function $v = 550 + 16.5t$, in which $t$ is the number of years the money is in the bank. What is the range of this function?

A $0 \leq v \leq 550$
B $v \geq 550$
C $v \leq 550$
D $0 \leq v \leq 16.5$

Answer Key: page 226
**Question 17**

John and some of his friends went waterskiing in his new boat. He made a table that showed the number of gallons of gas remaining at the end of each hour. The scatterplot below shows the gas that remained in terms of the hours that had passed.

Which of the following describes the correlation between the gas that remained and the hours that had passed?

A. Positive correlation  
B. No correlation  
C. Negative correlation  
D. Undefined correlation

**Question 18**

Alex and Millie are selling kites from their stand on the beach. Each day more people stop to look at and buy the kites. Alex and Millie kept the following record comparing the number of customers that stopped to look at their kites and the money they collected in sales each day.

<table>
<thead>
<tr>
<th>Kite Sales</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of customers</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
</tr>
<tr>
<td>Amount of sales</td>
<td>180</td>
<td>210</td>
<td>240</td>
<td>270</td>
</tr>
</tbody>
</table>

If this trend continues, how many customers will need to stop and look at Alex and Millie’s kites in order for their sales in one day to reach $330?

A. 42  
B. 60  
C. 66  
D. 48
**Question 19**

Frieda wants to buy a refrigerator that is 6 feet tall. The refrigerator's width is 1.75 times its depth. Which equation best describes \( V \), the volume of the refrigerator in terms of its depth, \( x \)?

A \( V = 6x + 10.5 \)
B \( V = x^2 + 1.75x \)
C \( V = 1.75x^2 + 6x \)
D \( V = 10.5x^2 \)

**Question 20**

The following ordered pairs belong to the function \( f(x) \).

\[(1, 2), (2, 8), (3, 18), (4, 32)\]

Which equation best describes this function?

A \( f(x) = x + 4 \)
B \( f(x) = 8x \)
C \( f(x) = 2x^2 \)
D \( f(x) = x - \frac{6}{x} \)

**Question 21**

Which algebraic expression best represents the relationship between the terms in the following sequence and \( n \), their position in the sequence?

1, 3, 5, 7, 9, ...

A \( n + 2 \)
B \( 2n + 1 \)
C \( 2n - 1 \)
D \( n + 1 \)
Question 22

Mr. Jones runs a bakery. His monthly profit in dollars, \( P(x) \), is given by the function \( P(x) = \frac{1}{2}x - 2 \), in which \( x \) represents the number of loaves of bread sold in a month. What is the minimum number of loaves he must sell next month if he is to have a profit of at least $3000?

Record your answer and fill in the bubbles. Be sure to use the correct place value.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

Question 23

Which of these is equivalent to the expression below?

\[ 2x + 2(3x - 4) + 3(8x - 4) \]

A \( \frac{8x - 5}{4} \)
B \( 32x - 8 \)
C \( 32x - 20 \)
D \( 4(8x + 5) \)
Objective 3

The student will demonstrate an understanding of linear functions.

For this objective you should be able to

● represent linear functions in different ways and translate among their various representations; and
● interpret the meaning of slope and intercepts of a linear function and describe the effects of changes in slope and y-intercept in real-world and mathematical situations.

What Is a Linear Function?

A linear function is any function whose graph is a line.

Is the function represented by the following table of values a linear function?

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td>−5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
</tr>
</tbody>
</table>

Graph the ordered pairs in the table on a coordinate grid.

The points lie on a line. Therefore, the function they represent is a linear function.
Try It

The table below describes a linear relationship.

\| x \| y \\
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>5/3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Does the equation $3y = 2x + 3$ represent the same linear function?

To confirm that the equation $3y = 2x + 3$ represents the same linear function as the table, substitute the ordered pairs from the table into the equation.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>Does $3y = 2x + 3$?</th>
<th>Yes/No</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>$3(<strong><strong>) \neq 2(</strong></strong>) + 3$</td>
<td>______</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$____ = ____$</td>
<td>______</td>
</tr>
<tr>
<td>1</td>
<td>5/3</td>
<td>$3(<strong><strong>) \neq 2(</strong></strong>) + 3$</td>
<td>______</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$____ = ____$</td>
<td>______</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$3(<strong><strong>) \neq 2(</strong></strong>) + 3$</td>
<td>______</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$____ = ____$</td>
<td>______</td>
</tr>
</tbody>
</table>

The table and the equation represent the same function.
What Is Slope?

The **slope** of a graph is its rate of change, how fast it increases or decreases. The slope of a line can also be described as its steepness, how fast the line rises or falls.

The rate of change of a line is the ratio that compares the change in $y$-values to the corresponding change in $x$-values for any two points on the graph.

A graph's slope is often described as its **rise** (change in $y$) over **run** (change in $x$). To find the slope of a line from its graph:

- Pick any two points on the graph.
- Find the change in $y$-values, $y_2 - y_1$, or the rise. Count the number of units up or down between the two points.
- Find the change in $x$-values, $x_2 - x_1$, or the run. Count the number of units left to right between them.
- Determine whether the slope is positive or negative. As you go from left to right:
  - if the line points up, the slope is positive.
  - if the line points down, the slope is negative.
- Write the slope as a ratio: \( \frac{\text{rise}}{\text{run}} \) or \( \frac{\text{change in } y}{\text{change in } x} \).
What is the slope of the line in the graph below?

Find the slope by counting the change in y-values and the corresponding change in x-values between any two points.

The y-value changes 2 units for every 5 units the x-value changes. As you go from left to right, the line points down, so the slope is negative.

The slope of the graph is \( \frac{\text{rise}}{\text{run}} = \frac{2}{5} \).

Another way to find the slope is to use the slope formula.

**Slope Formula**

For any two points \((x_1, y_1)\) and \((x_2, y_2)\) on a graph, the slope, \(m\), of the line that passes through them is:

\[
m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{change in y-values} \quad \text{change in x-values}
\]

What is the slope of the line passing through the points \((1, 1)\) and \((3, 5)\)?

Let \((x_1, y_1)\) be \((1, 1)\) and \((x_2, y_2)\) be \((3, 5)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{3 - 1} = \frac{4}{2} = 2
\]

The slope of the line is 2.
The set of ordered pairs below represents a linear function. What is the rate of change of the function?

\{(0, 1), (1, 3), (2, 5), (3, 7), (4, 9)\}

Since this is a linear function, you can choose any two ordered pairs belonging to the function to find its rate of change.

- Use the ordered pairs (0, 1) and (2, 5).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{2 - 0} = \frac{4}{2} = 2
\]

The rate of change of the function is 2.

- Use the ordered pairs (1, 3) and (4, 9).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 3}{4 - 1} = \frac{6}{3} = 2
\]

The rate of change of the function is still 2.

The rate of change, or slope, of a linear function does not depend on the points you pick to calculate the slope.

---

**Try It**

Suppose you graphed the number of miles you drove on a trip.

![Graph of distance vs. time](image)

Based on this graph, at what speed did you drive?

The slope of the graph represents your ______________, the number of miles per hour you drove.

To find the slope, compare the change in ______________ and ______________ between any two points on the graph.

You could use the following two points: (1, ______) and (______, 240).
The y-intercept of a graph is the y-coordinate of the point where the graph crosses the y-axis. The x-coordinate of any point on the y-axis is 0. Thus, the coordinates of the y-intercept are \((0, y)\).

**Objective 3**

The slope of the graph represents your speed, the number of miles per hour you drove. To find the slope, compare the change in y-values and x-values between any two points on the graph. You could use the following two points: \((1, 60)\) and \((4, 240)\).

\[
m = \frac{240 - 60}{4 - 1} = \frac{180}{3} = 60
\]

Your speed was 60 miles per hour.

**What Is the Slope-Intercept Form of a Linear Function?**

One form of the equation of a linear function is \(y = mx + b\). This form is called the slope-intercept form of a linear function.

When the equation of a linear function is written in the form \(y = mx + b\):

- \(m\) is the slope of the graph of the function; \(m\) is the value by which the x-term is being multiplied.
- \(b\) is the y-intercept of the graph of the function; \(b\) is the value that is being added to the x-term.

What are the slope and the y-intercept of the function \(y = 4x + 1\)?

The equation is in slope-intercept form, \(y = mx + b\). Read the values of \(m\) and \(b\) from the equation.

- The value by which the x-term is being multiplied is 4, so \(m = 4\).
- The value that is being added to the x-term is 1, so \(b = 1\).

The slope of the function, \(m\), is 4.
The y-intercept of the function, \(b\), is 1.
What are the slope and $y$-intercept of the function $6x + y = 10$?

To determine the slope and $y$-intercept, transform the equation $6x + y = 10$ into slope-intercept form, $y = mx + b$.

$6x + y = 10$

$\boxed{6x} + \boxed{y} = \boxed{10}$

$y = -6x + 10$

The equation is now in the form $y = mx + b$.

Read the values of $m$ and $b$ from the revised equation. For this function, $m = -6$, and $b = 10$.

The slope of the function, $m$, is $-6$.
The $y$-intercept of the function, $b$, is 10.

---

**Try It**

Find the slope and $y$-intercept of the graph of the linear function $3y = 6x - 9$.

To determine the slope and $y$-intercept of the graph, transform the equation $3y = 6x - 9$ into slope-intercept form, $y = mx + b$.

$3y = 6x - 9$

$\boxed{3y} = \boxed{6x} - \boxed{9}$

$y = \boxed{2x} - \boxed{3}$

Read the values of $m$ and $b$ from the revised equation. For this function, $m = 2$ and $b = -3$.
The slope of the function is 2.
The $y$-intercept of the function is $-3$. 

---
What Are the Effects on the Graph of a Linear Function If the Values of $m$ and $b$ Are Changed in the Equation $y = mx + b$?

In the equation $y = mx + b$, $m$ represents the slope of the graph, and $b$ represents the $y$-intercept of the graph. Changing either of these two constants, $m$ or $b$, will affect the graph of the function.

**Change in Slope, $m$**

<table>
<thead>
<tr>
<th>Function 1</th>
<th>Function 2</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \frac{1}{2}x$</td>
<td>$y = 3x$</td>
<td>Since $</td>
</tr>
<tr>
<td>$y = -3x + 1$</td>
<td>$y = -2x + 1$</td>
<td>Since $</td>
</tr>
</tbody>
</table>

The absolute value of a number indicates its distance from 0 on a number line. The symbol for the absolute value of $x$ is $|x|$. For example:

- $|-3| = 3$
- $|5| = 5$
- $|-8.2| = 8.2$
If the function $y = x - 6$ were changed to $y = 3x - 6$, what would be the effect on the graph of the function?

The equations are both in slope-intercept form, $y = mx + b$. In this form, $m$ represents the graph’s slope.

- The slope of the original function is 1.
- If the equation were changed to $y = 3x - 6$, the new value for $m$ would be 3.

The graph of the new function would be steeper because $|3| > |1|$. 

Do you see that . . .
Objective 3

Change in $y$-intercept, $b$

<table>
<thead>
<tr>
<th>Function 1</th>
<th>Function 2</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \frac{2}{3}x + 4$</td>
<td>$y = \frac{2}{3}x + 6$</td>
<td>Function 2 crosses the $y$-axis at a higher point.</td>
</tr>
<tr>
<td>$y = -x + 3$</td>
<td>$y = -x - 1$</td>
<td>Function 2 crosses the $y$-axis at a lower point.</td>
</tr>
</tbody>
</table>

If the $y$-intercept of the function $y = \frac{1}{2}x - 4$ were decreased by 3, what would be the equation of the new function?

The equation is in slope-intercept form, $y = mx + b$. In this form, $b$ represents the graph's $y$-intercept.

- The $y$-intercept of the given function is $-4$.
- If the $y$-intercept of the function were decreased by 3, it would be $-4 - 3 = -4 + (-3) = -7$. The new value for $b$ would be $-7$.

The equation of the new function would be $y = \frac{1}{2}x - 7$. 

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The function \( y = \frac{1}{4}x + 1 \) was changed to \( y = \frac{1}{4}x + 5 \). What is the effect on the graph of the function?

- The value for \( b \) in the first function is 1. Its graph crosses the \( y \)-axis at \((0, 1)\).
- The value for \( b \) in the second function is 5. Its graph crosses the \( y \)-axis at \((0, 5)\).

Since \( 5 - 1 = 4 \), the graph of \( y = \frac{1}{4}x + 5 \) is 4 units above the graph of \( y = \frac{1}{4}x + 1 \).

**Try It**

If the \( y \)-intercept of the function \( y = 2.5x - 1.75 \) were increased by 2.25, what would be the effect on the graph of the function?

The equation of the given function is in slope-intercept form.

In this form, _____ represents the slope of the line, and its \( y \)-intercept is _____.

If the \( y \)-intercept were increased by 2.25, the new \( y \)-intercept would be equal to _____ + _____ = _____.

The new line would cross the \( y \)-axis at a __________ point than the original line.

The new line would have the same slope, _____, as the original line.

The equation of the given function is in slope-intercept form. In this form, 2.5 represents the slope of the line, and its \( y \)-intercept is \(-1.75\). If the \( y \)-intercept were increased by 2.25, the new \( y \)-intercept would be equal to \(-1.75 + 2.25 = 0.5\). The new line would cross the \( y \)-axis at a higher point than the original line. The new line would have the same slope, 2.5, as the original line.
Objective 3

Slopes of lines can tell you whether the lines are parallel or perpendicular.

● If two lines are parallel, then they have the same slope, and their equations have the same value for $m$.

Look at the graphs of these two equations: $y = 3x - 5$ and $y = 3x + 2$.

![Parallel lines graph]

● If two lines are perpendicular, then they have negative reciprocal slopes, and their equations have negative reciprocal values for $m$.

Look at the graphs of these two equations: $y = -4x - 1$ and $y = \frac{1}{4}x - 1$.

![Perpendicular lines graph]
How Do You Write Linear Equations?

You can write linear equations in slope-intercept form, \( y = mx + b \), or in standard form, \( Ax + By = C \). In standard form, \( A, B, \) and \( C \) are integers, and \( A \) is usually greater than zero.

You can find the equation of a line given any of the following information:

- the slope and the y-intercept of the graph
- the slope and a point on the graph
- two points on the graph

Given the slope and the y-intercept

Identify the values for both \( m \), the slope, and \( b \), the y-intercept. Write the equation in slope-intercept form, \( y = mx + b \), using these values.

**What is the equation of the line with a slope of \( \frac{1}{3} \) and a y-intercept of \(-4\)?**

Find the values you should substitute into the equation \( y = mx + b \).

- If the slope is \( \frac{1}{3} \), then \( m = \frac{1}{3} \).
- If the y-intercept is \(-4\), then \( b = -4 \).

The equation of the line is:

\[
\begin{align*}
  y &= mx + b \\
  y &= \frac{1}{3}x - 4
\end{align*}
\]

**Try It**

Write the equation of the line with a slope of \(-2\) and a y-intercept of \( \frac{3}{5} \).

If the slope is \(-2\), then \( m = \boxed{\text{_____}} \).

If the y-intercept is \( \frac{3}{5} \), then \( b = \boxed{\text{_____}} \).

The equation of the function is:

\[
\begin{align*}
  y &= mx + b \\
  y &= \boxed{\text{_____}}x + \boxed{\text{_____}}
\end{align*}
\]

If the slope is \(-2\), then \( m = -2 \). If the y-intercept is \( \frac{3}{5} \), then \( b = \frac{3}{5} \).

The equation of the function is:

\[
y = -2x + \frac{3}{5}
\]
Given the slope and a point on the graph:

- Substitute the given values (the x- and y-coordinates of the given point and m, the slope) into the slope-intercept form of the equation, \( y = mx + b \).
- Solve the equation for \( b \).
- Substitute the values for \( m \) and \( b \) into the slope-intercept form, \( y = mx + b \).

What is the equation of a line with a slope of 9, passing through the point (3, 4)?

- Substitute \( x = 3 \), \( y = 4 \), and \( m = 9 \) into the slope-intercept form of the equation.
  
  \[
  y = mx + b \\
  4 = 9(3) + b
  \]

- Solve for \( b \).
  
  \[
  4 = 9(3) + b \\
  4 = 27 + b \\
  -23 = b
  \]

- Substitute the given value for \( m \), 9, and the value you found for \( b \), -23, into the slope-intercept form of the equation.

  \[
  y = mx + b \\
  y = 9x - 23
  \]

The equation of the line is \( y = 9x - 23 \).

This equation could also be written in standard form, \( Ax + By = C \), in which \( A \) is usually positive.

\[
\begin{align*}
  y &= 9x - 23 \\
  -9x &= -9x \\
  -9x + y &= -23
\end{align*}
\]

To change -9 to a positive value, multiply both sides of the equation by -1.

\( 9x - y = 23 \)

In standard form, the equation of this line is \( 9x - y = 23 \).
Try It
Write the equation of the linear graph that has a slope of 3 and contains the point (6, 5).
Begin by substituting the given values into the slope-intercept form.
The line passes through the point (____, ____). Therefore, 
\(x = \) _____ and \(y = \) _____.
Since the slope equals 3, \(m = \) _____.
\[ y = mx + b \]
\[ _____ = _____ \cdot _____ + b \]
Find the value for _____ that makes the equation true.
\[ _____ = _____ + b \]
\[ b = _____ \]
Finally, substitute the given value for \(m, \) _____, and the value you found for \(b, _____, \) into the slope-intercept form of the equation.
\[ y = mx + b \]
\[ y = _____x + _____ \]
\[ \text{or} \]
\[ y = _____x - _____ \]

Begin by substituting the given values into the slope-intercept form. The line passes through the point (6, 5). Therefore, \(x = 6\) and \(y = 5\). Since the slope equals 3, \(m = 3\).

\[ 5 = 3 \cdot 6 + b \]
Find the value for \(b\) that makes the equation true.
\[ 5 = 18 + b \]
\[ b = -13 \]
Finally, substitute the given value for \(m, 3,\) and the value you found for \(b, -13,\) into the slope-intercept form of the equation.
\[ y = 3x + (-13) \]
\[ \text{or} \]
\[ y = 3x - 13 \]
Given two points on the graph

- Use the x-coordinates and y-coordinates of the two given points to find the slope, \( m \), of the line.
- Substitute the coordinates of one of the known points and the slope you just found into the slope-intercept form of the equation, \( y = mx + b \).
- Solve the equation for \( b \).
- Substitute \( m \) and \( b \) into the slope-intercept form, \( y = mx + b \).

Find the equation of the line passing through the points (1, 2) and (5, 3).

- Find the slope of the graph using the slope formula. The line passes through the points (1, 2) and (5, 3).
  \[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{5 - 1} = \frac{1}{4}
\]
  If \( m \) is \( \frac{1}{4} \), the slope of the line is \( \frac{1}{4} \).
- Substitute the coordinates of either of the given points and the value you found for \( m \), \( \frac{1}{4} \), into the slope-intercept form of the equation. Find the value of \( b \).
  Use (1, 2) or (5, 3).

\[
\begin{align*}
  x = 1, & \quad y = 2 \\
  y = mx + b & \quad y = mx + b \\
  2 = \frac{1}{4}(1) + b & \quad 3 = \frac{1}{4}(5) + b \\
  2 = \frac{1}{4} + b & \quad 3 = \frac{5}{4} + b \\
  -\frac{1}{4} = -\frac{1}{4} & \quad -\frac{5}{4} = -\frac{5}{4} \\
  \frac{3}{4} = b & \quad \frac{3}{4} = b
\end{align*}
\]

- Substitute the values of \( m \) and \( b \) into the slope-intercept form of the equation.

\[
\begin{align*}
  y &= mx + b \\
  y &= \frac{1}{4}x + \frac{3}{4}
\end{align*}
\]

The equation of the line passing through points (1, 2) and (5, 3) is \( y = \frac{1}{4}x + \frac{3}{4} \).
Try It

Write the equation of the linear function that contains the points (4, 1) and (2, 3).

Find the __________ of the graph.

The line passes through the points (_____, _____) and (_____, _____).

The slope of the line is ______.

Substitute the coordinates of one of the given points, (4, ______), and the slope you found, ______, into the slope-intercept form of the equation of a line to find the value of \( b \).

\[
\begin{align*}
\text{y} &= \text{mx} + \text{b} \\
\text{_____} &= \text{_____} \cdot \text{_____} + \text{b} \\
\text{_____} &= \text{_____} + \text{b} \\
\text{b} &= \text{_____}
\end{align*}
\]

Substitute the values of \( m \) and \( b \) into the slope-intercept form.

\[
\text{y} = \text{_____} + \text{_____}
\]

Find the slope of the graph. The line passes through the points (4, 1) and (2, 3).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{2 - 4} = \frac{2}{-2} = -1
\]

The slope of the line is \(-1\).

Substitute the coordinates of one of the given points, (4, 1), and the slope you found, \(-1\), into the slope-intercept form of the equation of a line to find the value of \( b \).

\[
\begin{align*}
1 &= -1 \cdot 4 + b \\
1 &= -4 + b \\
b &= 5
\end{align*}
\]

Substitute the values of \( m \) and \( b \) into the slope-intercept form.

\[
\text{y} = -x + 5
\]
How Do You Find the x-intercept and y-intercept for an Equation?

Finding the x-intercept

The x-intercept is the point where the graph of a line crosses the x-axis. The x-intercept has the coordinates \((x, 0)\).

To find the x-intercept, substitute \(y = 0\) into the equation and solve for \(x\). The value of \(x\) is the x-intercept. The graph of the line crosses the x-axis at the point \((x, 0)\).

\[
\begin{align*}
\text{What is the x-intercept of the graph of } 5x - y &= 10? \\
\text{Substitute } y = 0 \text{ into the equation and solve for } x. \\
5x - 0 &= 10 \\
5x &= 10 \\
x &= 2 \\
\end{align*}
\]

The x-intercept of the graph is 2. The graph of the line crosses the x-axis at \((2, 0)\).

Finding the y-intercept

The y-intercept is the point where the graph of a line crosses the y-axis. The y-intercept has the coordinates \((0, y)\).

One way to find the y-intercept is to write the equation in slope-intercept form, \(y = mx + b\).

The value of \(b\) is the y-intercept. The graph of the line crosses the y-axis at the point \((0, b)\).

Another way to find the y-intercept is to substitute \(x = 0\) into the equation and solve for \(y\).

Here are two ways to find the y-intercept of the graph of \(y + 4 = 2x\).

Write the equation in slope-intercept form, \(y = mx + b\).

\[
\begin{align*}
y + 4 &= 2x \\
-4 &= -4 \\
y &= 2x - 4 \\
\end{align*}
\]

Therefore, \(b = -4\).

Substitute \(x = 0\) and solve for \(y\).

\[
\begin{align*}
y + 4 &= 2x \\
y + 4 &= 2(0) \\
y + 4 &= 0 \\
y &= -4 \\
\end{align*}
\]

The y-intercept of the graph is \(-4\). The graph crosses the y-axis at \((0, -4)\).
What are the x- and y-intercepts of the line of $4x + 3y = 24$?
To find the y-intercept, substitute $x = 0$ into the equation and solve for $y$.

$$4x + 3y = 24$$
$$4(0) + 3y = 24$$
$$0 + 3y = 24$$
$$3y = 24$$
$$y = 8$$

The y-intercept is 8.
To find the x-intercept, substitute $y = 0$ into the equation and solve for $x$.

$$4x + 3y = 24$$
$$4x + 3(0) = 24$$
$$4x + 0 = 24$$
$$4x = 24$$
$$x = 6$$

The x-intercept is 6.
How Do You Interpret the Meaning of Slopes and Intercepts?

To interpret the meaning of the slope or of the $x$- or $y$-intercept of a function in a real-life problem, follow these guidelines:

- The slope of the function is the function’s rate of change. A graph’s slope tells you how fast the function’s dependent variable is changing for every unit change in the independent variable.

For example, if a graph compares wages earned in dollars to hours worked, the slope tells the rate at which you are paid, or how much you make per hour.
The y-intercept is the point where the graph of a function crosses the y-axis. The y-intercept has the coordinates (0, y). It is the point in the function where the independent quantity, x, has a value of 0. The y-intercept is often the starting point in a problem situation.

For example, if a graph describes a constant temperature drop of 10°F per minute from \( t = 0 \) to \( t = 8 \) minutes, the y-intercept tells you what the temperature was at \( t = 0 \). The y-intercept of the graph is (0, 85), so the initial temperature was 85°F.
The x-intercept is the point where the graph of the function crosses the x-axis. The x-intercept has the coordinates (x, 0). It is the point in the problem where the dependent quantity, y, has a value of 0.

For example, if a graph compares the height of a falling object to the number of seconds it has fallen, the x-intercept (when the object’s height above the ground is 0) tells you how many seconds it will take for the object to hit the ground.

Gracie took a long driving trip. She started her journey at home and drove due west for two days. The graph below represents the number of miles Gracie was from her home on the second day of her trip. The slope of the graph is 60, and the y-intercept is (0, 200). Interpret the meaning of the slope and the y-intercept.

The slope of the graph is the function’s rate of change. It compares the number of miles Gracie was from home to the number of hours she was driving, or miles per hour. A slope of 60 means she was driving at the rate of 60 miles per hour, or that her speed was 60 mph.

The y-intercept is the point where the graph crosses the y-axis, when hours traveled equals 0. This point is the time when Gracie started driving on the second day. A y-intercept of 200 means she was already 200 miles from home when she began driving on the second day of her trip.
Try It
The graph below shows the weight of Denise's dog Elmo over the 6-month period after she adopted him. What was the dog's weight when she adopted him? How many pounds did he gain each month during that 6-month period?

The y-intercept of the graph is at the point (0, ____).
This means Elmo weighed _____ pounds when Denise adopted him.
The number of pounds Elmo gained each month is the graph's rate of ___________, or its ___________.
To find the slope of the graph, identify the coordinates of _____ points on the graph.
For example, (2, _____) and (______, 45) are points on the graph.
The slope of the graph is the change in the _____-coordinates between any two points compared to the corresponding change in their _____-coordinates.
The slope of the graph is \[
\frac{45 \,-\, \square}{4 \,-\, \square} = \square = \square.
\]
Elmo gained _____ pounds per month during the 6-month period.

The y-intercept of the graph is at the point (0, 25). This means Elmo weighed 25 pounds when Denise adopted him. The number of pounds Elmo gained each month is the graph's rate of change, or its slope. To find the slope of the graph, identify the coordinates of two points on the graph. For example, (2, 35) and (4, 45) are points on the graph. The slope of the graph is the change in the y-coordinates between any two points compared to the corresponding change in their x-coordinates. The slope of the graph is \[
\frac{45 \,-\, 35}{4 \,-\, 2} = \frac{10}{2} = 5.
\] Elmo gained 5 pounds per month during the 6-month period.
How Do You Predict the Effects of Changing Slopes and y-intercepts in Applied Situations?

Many real-life problems can be modeled with linear functions. To analyze such problems, it is often helpful to identify the slope and the y-intercept of the linear function. Interpreting the meaning of these values will help you predict the effect that changing them will have on the quantities in the problem.

- If the slope is changed, a rate of change in the problem will increase or decrease.
- If the y-intercept is changed, an initial condition will change.

Tess is filling the gas tank of her car. The graph represents the gallons of gas in her tank in terms of the number of minutes she pumps gas.

The graph crosses the y-axis at the point (0, 2.5). The graph suggests that Tess had 2.5 gallons of gasoline in her car when she started to pump gas.

If her car had had 7 gallons of gas in it when she started pumping gas, how would the graph be affected?

If her gas tank had had 7 gallons instead of 2.5 gallons when she started pumping gas, the graph would pass through the y-axis at the point (0, 7) instead of (0, 2.5).

The graphs would be parallel lines.
Try It

Zippy's, a package-delivery service, charges $12 plus $0.08 per mile to deliver a package within the city. How would the graph of the cost of delivering a package change if Zippy's increased its mileage charge to $0.09 per mile?

Write an equation that represents the cost, $c$, of delivering a package $n$ miles.

$$c = \underline{0.08} n + \underline{12}$$

In this equation 0.08 represents the slope of the graph of the equation.

If the mileage charge were changed from 0.08 to 0.09, the slope of the graph would increase. The new line will be steeper.

In this equation 0.08 represents the slope of the graph of the equation. If the mileage charge were changed from 0.08 to 0.09, the slope of the graph would increase. The new line will be steeper.
How Do You Solve Problems Involving Direct Variation or Proportional Change?

If a quantity $y$ varies directly with a quantity $x$, then the linear equation representing the relationship between the two quantities is $y = kx$. In this equation, $k$ is called the **proportionality constant**.

To say “$y$ varies directly with $x$” is to say “$y$ is directly proportional to $x$.”

If the equation $y = kx$ were graphed, $k$ would be the slope of the graph.

---

**The price of milk varies directly with the number of quarts of milk purchased. If 5 quarts of milk cost $6.25, what would 3 quarts of milk cost?**

- Write an equation that compares the number of quarts purchased to the cost.
  
  Let $q$ = the number of quarts purchased.
  
  Let $c$ = the cost.
  
  The direct variation equation is $c = kq$.

- Substitute the known values for $c$ and $q$ to find the proportionality constant, $k$. Five quarts of milk ($q = 5$) cost $6.25 ($c = 6.25$).
  
  
  $c = kq$
  
  $6.25 = k(5)$
  
  $1.25 = k$

  If $k = 1.25$, then the proportionality constant is $1.25$.

  If $k = 1.25$, then the slope of the graph is 1.25.

- Find the cost of 3 quarts of milk by substituting $k = 1.25$ and $q = 3$ into the equation $c = kq$.
  
  $c = kq$
  
  $c = 1.25(3)$
  
  $c = 3.75$

  Three quarts of milk cost $3.75.

---

**Now practice what you’ve learned.**
Question 24
Which of the following can best be described by a linear function?
A The area of a circle with radius $r$
B The perimeter of an equilateral triangle with side length $s$
C The surface area of a cube with side length $s$
D The volume of a cylinder with radius $r$ and height $h$

Question 25
Which graph best represents the equation $2y - x = 10$?
Question 26
Which of the following linear functions does not represent a rate of change of $\frac{2}{3}$?

A 2x + 3y = 12

B $2x + \frac{2}{3}y = 9$

C $\frac{2}{3}x + 9$

D $\frac{2}{3}x + 9$

Answer Key: page 228

Question 27
The graph below shows the number of grams of beef and the number of grams of potatoes you could eat to obtain approximately 500 calories of energy.

Which of the following numbers represents the maximum number of grams of potatoes you could eat to obtain approximately 500 calories?

A 200 g
B 450 g
C 100 g
D 150 g

Answer Key: page 228
Question 28
The graph projects a business’s growth in financial assets over a seven-year period.

Which of the following interpretations of the graph is true?

A The company’s initial assets are $200,000. The expected growth rate is $50 per year.
B The company’s initial assets are $200. The expected growth rate is $50,000 per year.
C The company’s initial assets are $200,000. The expected growth rate is $50,000 per year.
D The company’s initial assets are $200. The expected growth rate is $50 per year.

Question 29
Which of the following equations represents a graph that is parallel to the graph of the equation $8x + 2y = 10$ and that has a $y$-intercept of 8?

A $-4x + y = 5$
B $4x + y = 8$
C $8x - 2y = 10$
D $8x + 2y = 4$

Question 30
The line $y = \frac{3}{4}x - 4$ is drawn on a coordinate grid. A second line is drawn with a slope of 1.

Which statement best describes the relationship between these two graphs?

A The second line is steeper than the first line.
B The graphs are perpendicular lines.
C The second line is less steep than the first line.
D The graphs are parallel lines.

Question 31
Which equation is best represented by a line containing the points (2, −5) and (4, 3)?

A $x + 4y = 13$
B $y = 4x + 13$
C $y = -4x + 19$
D $-4x + y = -13$

Question 32
Find the coordinates of the $x$-intercept and the $y$-intercept of the line $2x = 9 - 3y$.

A $x$-intercept (3, 0); $y$-intercept $(0, \frac{9}{2})$
B $x$-intercept (0, 3); $y$-intercept $(\frac{9}{2}, 0)$
C $x$-intercept $(0, \frac{9}{2})$; $y$-intercept (3, 0)
D $x$-intercept $(\frac{9}{2}, 0)$; $y$-intercept (0, 3)
**Question 33**

A local bakery sells cookies by the dozen. If you want more than one dozen cookies, the bakery charges by the cookie for the additional cookies. The first graph below shows the original cost of buying a dozen or more cookies from the bakery. The second graph shows the cost of buying a dozen or more cookies after the bakery changed its prices.

Which statement describes how the bakery changed its price for a dozen or more cookies?

A  The bakery increased the cost of the first dozen cookies.
B  The bakery increased the cost per cookie after the first dozen.
C  The bakery decreased the cost per cookie after the first dozen.
D  The bakery decreased the cost of the first dozen cookies.

**Question 34**

The number of miles Sammie walks is directly proportional to the number of minutes she walks. If Sammie walks 3 miles in 45 minutes, what is the constant of proportionality, and how far would she walk in 2.5 hours?

A  $\frac{1}{15}$; 10 miles
B  $\frac{1}{15}$; 16.6 miles
C  15; 37.5 miles
D  15; 225 miles
Objective 4

The student will formulate and use linear equations and inequalities.

For this objective you should be able to
- formulate linear equations and inequalities from problem situations, use a variety of methods to solve them, and analyze the solutions; and
- formulate systems of linear equations from problem situations, use a variety of methods to solve them, and analyze the solutions.

How Do You Solve Problems Using Linear Equations or Inequalities?

Many real-life problems can be solved using either a linear equation or an inequality. To solve the equation or inequality, follow these steps:
- Simplify the expressions in the equation or inequality by removing parentheses and combining like terms.
- Isolate the variable as a single term on one side of the equation by adding or subtracting expressions on both sides of the equation or inequality.
- Use multiplication or division to produce a coefficient of 1 for the variable term.
- When solving an inequality, you must reverse the inequality symbol if you multiply or divide both sides by a negative number.

\[ -2x > 10 \]
\[ \frac{-2x}{-2} < \frac{10}{-2} \]
\[ x < -5 \]

The inequality symbol reversed; it went from \( > \) to \( < \).
- Use the solution of the equation or inequality to find the answer to the question asked.
- See whether your answer is reasonable.

The combined weight of 3 people in a small plane cannot safely exceed 620 pounds. The pilot weighs 185 pounds, and Ramon weighs \( \frac{3}{2} \) times as much as his wife Grace. Altogether they do not exceed the weight limit. Write an inequality that could be used to find Grace’s maximum possible weight.
- Represent the quantities involved with variables or expressions. You know that Ramon’s weight is 1.5 times Grace’s weight. Represent Grace’s weight with \( g \).
- Ramon weighs 1.5 times his wife’s weight, or 1.5\( g \).
Objective 4

Write the inequality.
Grace's weight + Ramon's weight + the pilot's weight must be
less than or equal to 620 pounds.
\[ g + 1.5g + 185 \leq 620 \]
\[ 1g + 1.5g + 185 \leq 620 \]
\[ 2.5g + 185 \leq 620 \]
The inequality \( 2.5g + 185 \leq 620 \) could be used to find Grace's
maximum possible weight.

In the school Adopt-a-Highway program, Trent picked up twice as
many empty soda cans as Susan did but only one-third as many as
Ginger collected. Together, the team picked up 135 cans. How
many cans did Trent pick up?

- Represent the unknown quantities with variables or expressions.
The number of cans each team member picked up is described
in terms of the number of cans Trent picked up. Represent the
number of cans Trent picked up using the variable \( t \).
  - Susan picked up \( \frac{1}{2} \) as many cans as Trent.
  - Ginger picked up 3 times as many cans as Trent.

<table>
<thead>
<tr>
<th>Trent</th>
<th>Susan</th>
<th>Ginger</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( 0.5t )</td>
<td>( 3t )</td>
</tr>
</tbody>
</table>

- Write an equation that can be used to solve the problem.
Together the team picked up 135 cans.
\[ t + 0.5t + 3t = 135 \]

- Simplify the expression by combining like terms. Then solve
the equation for \( t \).
\[ 1t + 0.5t + 3t = 135 \]
\[ (1 + 0.5 + 3)t = 135 \]
\[ 4.5t = 135 \]
\[ t = 30 \]

In this problem, the solution to the equation, \( t = 30 \), is also the
answer to the problem. Trent picked up 30 empty soda cans.
The width of a rectangle is between 5 and 10 inches. If its length is 1 inch more than twice its width, what is a reasonable range for the rectangle's perimeter?

Its width can be represented by \( w \).
Its length can be represented by \( 2w + 1 \).
Its perimeter can be represented by \( P \).

\[
P = 2l + 2w
P = 2(2w + 1) + 2w
P = 4w + 2 + 2w
P = 6w + 2
\]

Find a reasonable range for \( P \).

Substitute \( w = 5 \) \( w = 10 \)

\[
(6w + 2) < P < (6w + 2)
(6 \cdot 5 + 2) < P < (6 \cdot 10 + 2)
(30 + 2) < P < (60 + 2)
32 < P < 62
\]

Any perimeter between 32 inches and 62 inches would be reasonable.

An illustrator wants to produce a series of pen-and-ink drawings. The height of each drawing must be at least 1 inch more than \( \frac{1}{3} \) its width. This relationship is represented in the graph below.

If a drawing 6 inches wide is 5 inches tall, does it meet the requirements?

Find the 6 on the \( x \)-axis and then go up to the shaded region in the graph. Is the point (6, 5) in the shaded region? Yes.

Then a 6-inch-by-5-inch drawing meets the requirements.
Objective 4

Try It

The length of a rectangle is 5 inches greater than its width. If the width of the rectangle is represented by \( w \) and its perimeter is represented by \( P \), write an equation in terms of \( P \) and \( w \) that could be used to find the dimensions of the rectangle.

Represent the width of the rectangle by \( w \).

The length of the rectangle is 5 inches greater than its width.

Represent the length by \( w + \) _______.

Write an equation and simplify it.

\[
P = 2l + 2w
\]
\[
P = 2(w + \text{_______}) + 2w
\]
\[
P = 2w + \text{_______} + 2w
\]
\[
P = \text{_______}w + \text{_______}
\]

The equation \( P = \text{_______}w + \text{_______} \) could be used to find the dimensions of the rectangle.

The length of the rectangle is 5 inches greater than its width. Represent the length by \( w + 5 \).

\[
P = 2l + 2w
\]
\[
P = 2(w + 5) + 2w
\]
\[
P = 2w + 10 + 2w
\]
\[
P = 4w + 10
\]

The equation \( P = 4w + 10 \) could be used to find the dimensions of the rectangle.
Try It

Saul used two different types of trim molding on a woodworking project. He used 6 feet more of the molding that cost $2.50 per foot than he used of the molding that cost $1.50 per foot. If the combined cost of the molding used to complete his project was $55, how many feet of each type of molding did Saul use?

Represent the number of feet of $1.50-per-foot molding with $m$.

Represent the number of feet of $2.50-per-foot molding with $m + 6$.

Represent the cost of the $1.50 molding used.

$1.50m$

Represent the cost of the $2.50 molding used.

$2.50(m + 6)$

Write an equation that shows the total cost as $55 and solve for $m$.

$1.50m + 2.50(m + 6) = 55$

$1.50m + 2.50m + 15 = 55$

$m = 10$

Saul used 10 feet of molding that cost $1.50 per foot. He used $m + 6 = 10 + 6 = 16$ feet of molding that cost $2.50 per foot.
Try It

A farmer sells peaches and decorative baskets at a roadside stand. He sells the baskets for $4.95 each and the peaches for $0.69 per pound. The equation $p = 0.69w + 4.95$ represents the price, $p$, of a basket containing $w$ pounds of peaches. The graph of this relationship is shown below.

Use the graph to find a reasonable value for the number of pounds of peaches in a decorative basket that would sell for $10.00 altogether.

Go to the point on the ________________ axis where the cost is approximately $10.00.

Go __________________ until you reach the graph.

Go __________________ to the ________________ axis and read the value there.

The corresponding value is greater than __________.

A reasonable value for the number of pounds of peaches in a basket costing $10.00 is about __________ pounds.

Go to the point on the vertical axis where the cost is approximately $10.00.

Go across until you reach the graph. Go down to the horizontal axis and read the value there. The corresponding value is greater than 7. A reasonable value for the number of pounds of peaches in a basket costing $10.00 is about $7\frac{1}{3}$ pounds.
**How Do You Represent Problems Using a System of Linear Equations?**

Many real-life problems can be solved using a system of two or more linear equations.

To represent a problem using a system of linear equations, follow these guidelines.

- Identify the quantities involved and the relationships between them.
- Represent the quantities involved with two different variables or with expressions involving two variables.
- Write two independent equations that can be used to solve the problem.

The sum of two numbers is 100. Twice the first number plus three times the second number is 275. Write a system of two equations in two unknowns that could be used to find the two numbers.

The problem involves two numbers. One number is not described in terms of the other number, so it makes sense to represent them using two different variables.

- Represent the first number with the variable $x$.
- Represent the second number with the variable $y$.

You know two different relationships between the numbers.

- Their sum is 100.
  $$x + y = 100$$
- Twice the first number plus three times the second number is 275.
  $$2x + 3y = 275$$

The following system of linear equations could be used to find the two numbers.

$$\begin{align*}
x + y &= 100 \\
2x + 3y &= 275
\end{align*}$$
Try It

Daniel put all the pennies and nickels from his pocket change into a jar. At the end of the month, there were 210 coins in his jar, worth $3.30 in all. Write a system of two equations that can be used to find \( p \), the number of pennies, and \( n \), the number of nickels, in the jar at the end of the month.

The value of a group of mixed coins depends on the value of each type of coin.

The number of any one type of coin times its ______________ gives the total value of all the coins of that type.

Represent the number of coins of each type Daniel had in his jar.
Represent the number of pennies with the variable \( p \).
Represent the number of nickels with the variable \( n \).
Represent the total value in cents of each type of coin in his jar.

Represent the value of the pennies with the expression _________.
Represent the value of the nickels with the expression _________.
Write two equations that describe the relationships between these quantities:
The number of pennies + the number of nickels = ________ coins in the jar:
________ + ________ = ________
The value of the pennies + the value of the nickels = ________ cents in the jar:
________ + ________ = ________
The following system of equations could be used to find the number of each type of coin Daniel has in his jar:

\[ p + n = \]
\[ _____p + _____n = \]
Objective 4

101

The solution to a system of linear equations is a pair of numbers that makes both equations true. For example, the ordered pair (4, 2) is a solution to the following system of equations:

\[
\begin{align*}
2x + y &= 10 \\
-x + y &= 2
\end{align*}
\]

because when \( x = 4 \) and \( y = 2 \), each equation is true.

The number of any one type of coin times its value gives the total value of all the coins of that type. Represent the value of the pennies with the expression \( 1p \). Represent the value of the nickels with the expression \( 5n \). The number of pennies + the number of nickels = 210 coins in the jar: \( p + n = 210 \). The value of the pennies + the value of the nickels = 330 cents in the jar: \( 1p + 5n = 330 \). The following system of equations could be used to find the number of each type of coin Daniel has in his jar:

\[
\begin{align*}
p + n &= 210 \\
1p + 5n &= 330
\end{align*}
\]

How Do You Solve a System of Linear Equations?

You can solve a system of linear equations algebraically and graphically. Two algebraic methods are substitution and elimination.

A graph can show you how many solutions a system of equations has.

<table>
<thead>
<tr>
<th>If the two lines ...</th>
<th>... as shown by this graph ...</th>
<th>... then the system of equations ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>intersect at a single point</td>
<td><img src="image" alt="Graph" /></td>
<td>has one solution: ( x = 1 ) and ( y = 2 ) or ( (1, 2) ).</td>
</tr>
<tr>
<td>do not intersect (parallel lines)</td>
<td><img src="image" alt="Graph" /></td>
<td>has no solution.</td>
</tr>
<tr>
<td>intersect at every point (the same line)</td>
<td><img src="image" alt="Graph" /></td>
<td>has many solutions.</td>
</tr>
</tbody>
</table>
You can also determine the number of solutions a system of equations has by writing the equations in slope-intercept form, \( y = mx + b \).

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Have different values for ( m )</td>
<td>( 2x + y = 4 )</td>
<td>( y = -2x + 4 )</td>
<td>Has one solution: ( x = 1 ) and ( y = 2 ).</td>
</tr>
<tr>
<td>Have different values for ( b ) but the same value for ( m )</td>
<td>( 2x + y = 4 )</td>
<td>( y = -2x + 4 (b = 4) )</td>
<td>Has no solution.</td>
</tr>
<tr>
<td>Have the same values for ( m ) and ( b )</td>
<td>( 2x + y = 4 )</td>
<td>( y = -2x + 4 )</td>
<td>Has many solutions.</td>
</tr>
</tbody>
</table>

Solve this system of equations using the graphical method.

\[-x + y = -2\]
\[2x - y = 5\]

The two equations are graphed below.

The coordinates of the point where the two lines intersect, \((3, 1)\), is the solution to the system of equations. The coordinates satisfy both equations.

<table>
<thead>
<tr>
<th>( x = 3 ) and ( y = 1 )</th>
<th>( -x + y = -2 )</th>
<th>( 2x - y = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 3 )</td>
<td>( -x + y = -2 )</td>
<td>( 2x - y = 5 )</td>
</tr>
<tr>
<td>( -3 ) + ( 1 ) = (-2)</td>
<td>( 2(3) - (1) = 5 )</td>
<td>( 5 = 5 )</td>
</tr>
</tbody>
</table>

The solution to the system of equations is \((3, 1)\).
When you use the substitution method to solve a system of equations, solve one equation for one of the two variables. Then substitute into the second equation.

Solve the following system of equations using the substitution method.

\[ \begin{align*}
  a + b & = 30 \\
  2a - 3b & = 10
\end{align*} \]

This system of equations lends itself to being solved using substitution because the first equation can be solved easily for \( a \) in terms of \( b \).

To solve the first equation for \( a \), subtract \( b \) from both sides.

\[ \begin{align*}
  a + b & = 30 \\
  - b & = - b \\
  a & = 30 - b
\end{align*} \]

Now substitute \((30 - b)\) for \( a \) in the second equation.

The result will be a linear equation with just one variable.

Solve that equation.

\[ \begin{align*}
  2a - 3b & = 10 \\
  2(30 - b) - 3b & = 10 & \text{Substitute } (30 - b) \text{ for } a. \\
  60 - 2b - 3b & = 10 & \text{Remove parentheses; multiply by 2.} \\
  60 - 5b & = 10 & \text{Combine like terms: } -2b - 3b = -5b. \\
  60 & = 10 + 5b & \text{Add } 5b \text{ to both sides.} \\
  50 & = 5b & \text{Subtract 10 from both sides.} \\
  10 & = b & \text{Divide both sides by } 5.
\end{align*} \]

Now that you know the value of \( b \), you can use this value to find the value of \( a \) by substituting the value of \( b \) into either of the two original equations.

In this system, the first equation is the easier equation to use.

\[ \begin{align*}
  a + b & = 30 \\
  a + 10 & = 30 & \text{Substitute } b = 10. \\
  a & = 20 & \text{Subtract 10 from both sides.}
\end{align*} \]

The solution to the system of equations is \( a = 20 \) and \( b = 10 \), or \((20, 10)\).
The elimination method is also called the addition method because you eliminate one of the variables by adding. Before you add, you may need to multiply one or both equations so that one of the variables has opposite coefficients.

Solve the same system of equations using the elimination method.

\[ a + b = 30 \]
\[ 2a - 3b = 10 \]

In the given system of equations, if you multiply both sides of the first equation by 3, you get the following equations:

\[ 3(a + b) = 3(30) \quad \Rightarrow \quad 3a + 3b = 90 \]
\[ 2a - 3b = 10 \quad \Rightarrow \quad 2a - 3b = 10 \]

The variable \( b \) has a coefficient of 3 in the new equation and an opposite coefficient of \(-3\) in the original second equation. When you add those two equations, the term containing \( b \) will disappear because it will have a 0 coefficient.

\[
\begin{align*}
3a + 3b &= 90 \\
+ 2a - 3b &= 10 \\
5a &= 100 \\
a &= 20
\end{align*}
\]

To find \( b \), substitute 20 for \( a \) into the first equation.

\[
\begin{align*}
a + b &= 30 \\
20 + b &= 30 \\
b &= 10
\end{align*}
\]

The solution to the system of equations is \( a = 20 \) and \( b = 10 \), or \((20, 10)\). This is the same solution obtained by using the substitution method.
**Try It**

At the concession stand a can of soda costs $0.25 more than a bottle of water. If John bought 3 bottles of water and 2 cans of soda for $8, how much did each type of drink cost?

Let \( s \) represent the cost of a can of soda.
Let \( w \) represent the cost of a bottle of water.

If a can of soda costs $0.25 more than a bottle of water, then an equation that can be used to represent this is

\[
\begin{align*}
\textbf{Equation 1:} & \quad s = w + 0.25 \\
\textbf{Equation 2:} & \quad 3w + 2s = 8.00
\end{align*}
\]

Solve the system of equations using the substitution method.
Substitute \( w + 0.25 \) for \( s \) in the second equation and solve.

\[
\begin{align*}
3w + 2(w + 0.25) &= 8.00 \\
3w + 2w + 0.50 &= 8.00 \\
5w &= 7.50 \\
w &= 1.50
\end{align*}
\]

A bottle of water costs $1.50.
A can of soda costs \( s = w + 0.25 = 1.50 + 0.25 = \$1.75 \).

Now practice what you’ve learned.
**Question 35**
Jeanne wants to build a fence to enclose an area for a new rose garden. She can afford 150 feet of fencing. The length of the rectangular garden will be 5 feet more than the width, \( w \). Which inequality best describes the possible width of her garden?

A  \[ w + (w + 5) \leq 150 \]
B  \[ 2w + 2(w + 5) = 150 \]
C  \[ 2w + 2(w + 5) \leq 150 \]
D  \[ w + (w + 5) \geq 150 \]

**Question 36**
Sharon kept track of her expenses last week. She spent $4 more on movie rentals than she did on lunches. She spent five times as much fixing her car as she did on movie rentals. If Sharon spent a total of $80 last week on these three expenses, how much did her car repairs cost?

A  $8
B  $12
C  $60
D  $14

**Question 37**
The sum of two numbers is 59. The difference between 2 times the first number and 6 times the second is –34. Find the two numbers.

A  40 and 19
B  38 and 97
C  –38 and 97
D  –40 and –19

**Question 38**
Each morning at his bagel shop, Sid makes at least three times as many plain bagels as he does onion bagels and 25 more onion bagels than garlic bagels. The graph below represents the relationship between the number of plain bagels, \( p \), and the number of garlic bagels, \( g \), Sid prepares each day.

Which statement below does not satisfy this inequality relationship?

A  Sid made 100 garlic bagels and 390 plain bagels.
B  Sid made 50 garlic bagels and 225 plain bagels.
C  Sid made 25 garlic bagels and 125 plain bagels.
D  Sid made 75 garlic bagels and 300 plain bagels.
Question 39
Ira wants to build a rectangular dog kennel adjacent to the back wall of his garage.

Using the garage wall as the fourth side of the kennel allows him to fence only three sides of the kennel. The fencing material he is using costs $4 per foot. Ira has $120 to spend on the project. If the back wall of the garage is 20 feet long, what is the maximum width Ira can make the kennel?

A 10 ft  
B 60 ft  
C 100 ft  
D 5 ft

Question 40
Look at the following system of equations.

\[ 3x - 2y = 14 \]
\[ 6x - 2y = 32 \]

Which of the following is a description of the solution for this system?

A An infinite number of solutions  
B No solution  
C One solution  
D Two solutions

Question 41
Sandra spent $48.40 on tickets for a movie sneak preview. She bought 3 adult tickets and 5 child tickets. If the cost of an adult ticket, \( a \), is twice as much as the cost of a child ticket, \( c \), what is the cost of each kind of ticket?

A  \( a = 8.80 \)  
   \( c = 4.40 \)  
B  \( a = 13.82 \)  
   \( c = 6.91 \)  
C  \( a = 4.40 \)  
   \( c = 2.20 \)  
D  \( a = 6.91 \)  
   \( c = 3.46 \)

Question 42
An ice-cream store projects that the profit, \( p \), it earns on a total sales volume of \( s \) dollars is given by the formula \( p = 0.25(s - 3000) \). If sales for the next month are projected to be between $5000 and $7000, what range best represents the total profit the store can expect for that month?

A  \( 500 \leq p \leq 1000 \)  
B  \( 5000 \leq p \leq 7000 \)  
C  \( 0 \leq p \leq 2000 \)  
D  \( 2000 \leq p \leq 4000 \)
Objective 4

MATHEMATICS

Question 43

The members of a school choir had a fund-raising drive last month. They sold candy bars for $2 each and cans of popcorn for $5 each. Brent sold more than $300 worth of candy and popcorn altogether. Brent’s sales can be represented by the inequality $5x + 2y > 300$.

Which of the following points could not reasonably represent the number of candy bars and cans of popcorn sold by Brent last month?

A. (30, 90)
B. (40, 80)
C. (20, 50)
D. (50, 40)

Question 44

The Beachfront Resort charges its guests according to the number of nights they stay at the resort and the number of meals they eat there. A guest can stay 2 nights and have 5 meals for $395, or a guest can stay 5 nights and have 11 meals for $959. Which system of equations can be used to find $n$, the cost of a night’s stay, and $m$, the cost of a meal?

A. $5n + 2m = 395$
   $11n + 5m = 959$
B. $5n + 2m = 959$
   $11n + 5m = 395$
C. $2n + 5m = 959$
   $5n + 11m = 395$
D. $2n + 5m = 395$
   $5n + 11m = 959$

Question 45

Rachel is selling watermelons for $2 each and cantaloupes for $1 each. A customer bought a total of 13 watermelons and cantaloupes for $20. Which system of equations best describes the number of watermelons, $w$, and the number of cantaloupes, $c$, the customer bought?

A. $w + c = 20$
   $w + 2c = 13$
B. $w + c = 13$
   $w + 2c = 20$
C. $w + c = 20$
   $2w + c = 13$
D. $w + c = 13$
   $2w + c = 20$
Objective 5

The student will demonstrate an understanding of quadratic and other nonlinear functions.

For this objective you should be able to

- interpret and describe the effects of changes in the parameters of quadratic functions;
- solve quadratic equations using appropriate methods; and
- apply the laws of exponents in problem-solving situations.

What Is a Quadratic Function?

- A quadratic function is any function whose graph is a parabola.
- A quadratic equation is any equation that can be written in the form $y = ax^2 + bx + c$. The constants $a$, $b$, and $c$ are called the parameters of the equation. When you know their values, they help you describe the shape and location of the parabola.

The simplest quadratic function is $y = x^2$. It is the quadratic parent function.

The graph of the quadratic function $y = x^2 - 4$ is shown below.
What Happens to the Graph of $y = ax^2$ When $a$ Is Changed?

If two quadratic functions of the form $y = ax^2$ differ only in the sign of the coefficient of $x^2$, then one graph will be a reflection of the other graph across the $x$-axis.

- If $a > 0$, then the parabola opens upward.
- If $a < 0$, then the parabola opens downward.

How do the graphs of $y = 3x^2$ and $y = -3x^2$ compare?

In one function, $a = 3$. In the other function, $a = -3$. Each graph is a reflection of the other across the $x$-axis.

The graph of $y = 3x^2$ opens upward because $3 > 0$.

The graph of $y = -3x^2$ opens downward because $-3 < 0$. 

![Graph of $y = 3x^2$ and $y = -3x^2$](image)
If two quadratic functions of the form $y = ax^2$ have different coefficients of $x^2$, then one graph will be wider than the other. The smaller the absolute value of $a$, the coefficient of $x^2$, the wider the graph.

Which of these three functions produces the narrowest graph? 

$y = \frac{2}{3}x^2 \quad y = 2x^2 \quad y = -4x^2$

Compare the absolute value of the three coefficients of $x^2$.

- In the first function the coefficient of $x^2$ is $\frac{2}{3}$.
  In the second function the coefficient of $x^2$ is 2.
  In the last function it is $-4$.
- Since $|\frac{2}{3}| = \frac{2}{3}$, then $\frac{2}{3}$ has the least absolute value. The graph of $y = \frac{2}{3}x^2$ produces the widest parabola.
- Since $|-4| = 4$, then $-4$ has the greatest absolute value. The graph of $y = -4x^2$ produces the narrowest parabola.
If the coefficient of \( x^2 \) in the function \( y = x^2 \) is changed to \(-2\), what is the effect on the graph?

First find the effect of changing the sign of \( a \). Then find the effect of changing its value.

- If the coefficient of \( x^2 \) changes from positive 1 to negative 1, the new graph will be a reflection of the original graph. The graph of \( y = -x^2 \) will be a reflection of the graph of \( y = x^2 \) across the \( x \)-axis.

- If the coefficient of \( x^2 \) changes from \(-1\) to \(-2\), the graph becomes narrower because \(|-2| > |-1|\). The graph of \( y = -2x^2 \) will be narrower than the graph of \( y = -1x^2 \).

The graph of \( y = -2x^2 \) is narrower than the graph of \( y = x^2 \), and it opens down, not up.
Two quadratic functions are graphed below.

\[ y = \frac{1}{3}x^2 \quad \text{and} \quad y = 5x^2 \]

Which parabola is the graph of \( y = \frac{1}{3}x^2 \)?
Which parabola is the graph of \( y = 5x^2 \)?

Compare the absolute values of the coefficients of \( x^2 \).
- Since \( \frac{1}{3} \) is the smaller value, \( y = \frac{1}{3}x^2 \) produces the wider graph.
- Since 5 is the greater value, \( y = 5x^2 \) produces the narrower graph.

Parabola \( r \) is the graph of \( y = \frac{1}{3}x^2 \), and parabola \( t \) is the graph of \( y = 5x^2 \).
What Happens to the Graph of \( y = x^2 + c \) When \( c \) Is Changed?
If two quadratic functions of the form \( y = x^2 + c \) have different constants, \( c \), then one graph will be a translation up or down of the other graph.

How does the graph of \( y = x^2 + 4 \) compare to the graph of \( y = x^2 \)?
In the function \( y = x^2 + 4 \), the constant 4 has been added to the parent function \( y = x^2 \).

The graph of \( y = x^2 \) has been translated up 4 units.

How does the graph of \( y = x^2 - 2 \) compare to the graph of \( y = x^2 \)?
In the function \( y = x^2 - 2 \), the constant \(-2\) has been added to the parent function \( y = x^2 \).

The graph of \( y = x^2 \) has been translated down 2 units.
How many units apart are the vertices of the graphs of \( y = x^2 + 3 \) and \( y = x^2 - 1 \)?

- Adding the constant 3 to the parent function \( y = x^2 \) causes the parent function to be translated 3 units up.
- Adding the constant -1 to the parent function \( y = x^2 \) causes the parent function to be translated 1 unit down.
- Look at the graphs of the two functions.

The vertex of the graph of \( y = x^2 + 3 \) is 4 units higher than the vertex of the graph of \( y = x^2 - 1 \).
**Try It**

The graph of the function \( y = x^2 - 2 \) is translated 5 units up. The equation \( y = x^2 - 2 \) and the translated function are graphed below.

What is the equation of the translated graph?

In the equation \( y = x^2 - 2 \), \( c = \) ________.

If the graph is translated 5 units up, then the value of \( c \) increases ________ units.

The value of \( c \) goes from ________ to ________.

The equation of the translated function is ____________________.

---

In the equation \( y = x^2 - 2 \), \( c = -2 \). If the graph is translated 5 units up, then the value of \( c \) increases 5 units. The value of \( c \) goes from \(-2\) to \(+3\). The equation of the translated function is \( y = x^2 + 3 \).
How Do You Draw Conclusions from the Graphs of Quadratic Equations?

To analyze graphs of quadratic equations and draw conclusions from them, consider the following.

- Understand the problem. Identify the quantities involved and the relationship between them.
- Identify the quantities represented on the graph by using the horizontal and vertical axes and looking at the scales used.
- Find the x-intercepts and y-intercept of the graph and determine what these values represent in the problem.
- Decide whether the graph has a minimum or maximum point and determine what this value represents in the problem.

A golf ball was hit into the air. The graph below shows the height of the ball $t$ seconds after it was hit.

What conclusions about the ball’s path can you draw from the graph?

The horizontal axis represents time in seconds. The vertical axis represents the height of the ball in feet.

- The point $(0, 0)$ on the graph tells you that at 0 seconds the height of the ball was 0 feet.
- The greatest value of the function is at the vertex, $(3, 144)$. This means that the maximum height of the ball was 144 feet and that it took 3 seconds for the ball to reach this height.
- The point $(6, 0)$ on the graph tells you that at 6 seconds the ball was back on the ground, at 0 feet. The ball was in the air a total of 6 seconds.
If a stone is dropped from any height, its distance, \( d \), from the ground is modeled by the quadratic equation \( d = -16t^2 + h \), where \( t \) is the number of seconds it falls and \( h \) is the height in feet from which it was dropped.

The graph below models the distance from the ground of a stone dropped from the top of a tall building.

From what height was the stone dropped?
The stone was dropped when \( t = \) __________.
On this graph, \( t = 0 \) at the point (0, __________).
The stone was dropped from a height of __________ feet.

The stone was dropped when \( t = 0 \). On this graph, \( t = 0 \) at the point (0, 275). The stone was dropped from a height of 275 feet.
How Can You Solve a Quadratic Equation Graphically?

To find solutions to the quadratic equation \( ax^2 + bx + c = 0 \), you can look at the graph of the related quadratic function, \( y = ax^2 + bx + c \).

A quadratic equation can have 0, 1, or 2 solutions. The number of solutions is shown by the graph of the related quadratic function.

The solutions to a quadratic function are the graph's \( x \)-intercepts, the \( x \)-coordinates of the points where the graph crosses the \( x \)-axis.

The solutions can also be called:
- the roots of the quadratic function
- the zeros of the quadratic function
What are the solutions to the equation \( x^2 + 3x - 4 = 0 \)?

The graph of \( y = x^2 + 3x - 4 \) is shown below.

- The points where the graph crosses the \( x \)-axis are the points \((1, 0)\) and \((-4, 0)\). The \( x \)-coordinates of these points are 1 and \(-4\).
- The roots, or the zeros, of the function \( y = x^2 + 3x - 4 \) are 1 and \(-4\).
- Therefore, the solution set of the equation \( x^2 + 3x - 4 = 0 \) is \([-4, 1]\).

You can verify that these numbers are solutions by replacing \( x \) with their value in the quadratic equation.

\[
\begin{align*}
\text{Substitute } x &= 1 & \text{Substitute } x &= -4 \\
\quad x^2 + 3x - 4 &= 0 & \quad x^2 + 3x - 4 &= 0 \\
\quad (1)^2 + 3(1) - 4 &= 0 & \quad (-4)^2 + 3(-4) - 4 &= 0 \\
\quad 1 + 3 - 4 &= 0 & \quad 16 - 12 - 4 &= 0 \\
\quad 0 &= 0 & \quad 0 &= 0
\end{align*}
\]

Both numbers make the equation true. Both 1 and \(-4\) are solutions.
How Can You Solve a Quadratic Equation by Using a Table?

You can use the values representing a quadratic function in a table to find solutions to a quadratic equation.

- Identify the points in the table that have $y$-values of 0.
- The $x$-values of those points are the solutions to the equation.

The table below models the function $f(x) = x^2 + 6x + 5$. Find solutions to the equation $x^2 + 6x + 5 = 0$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>−1</td>
<td>0</td>
</tr>
<tr>
<td>−2</td>
<td>−3</td>
</tr>
<tr>
<td>−3</td>
<td>−4</td>
</tr>
<tr>
<td>−4</td>
<td>−3</td>
</tr>
<tr>
<td>−5</td>
<td>0</td>
</tr>
<tr>
<td>−6</td>
<td>5</td>
</tr>
</tbody>
</table>

The roots of the function are the $x$-coordinates of the points where the $y$-coordinate is 0.

Look for any points in the table where the $y$-coordinate is 0.

The points $(-1, 0)$ and $(-5, 0)$ are points where the $y$-coordinate is 0. The $x$-values of these points are $-1$ and $-5$.

The solution set of the quadratic equation $x^2 + 6x + 5 = 0$ is $\{-5, -1\}$.

You can confirm that these are the solutions by replacing $x$ with these values in the equation $x^2 + 6x + 5 = 0$.

<table>
<thead>
<tr>
<th>Substitute $x = -1$</th>
<th>Substitute $x = -5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 6x + 5 = 0$</td>
<td>$x^2 + 6x + 5 = 0$</td>
</tr>
<tr>
<td>$(-1)^2 + 6(-1) + 5 \neq 0$</td>
<td>$(-5)^2 + 6(-5) + 5 \neq 0$</td>
</tr>
<tr>
<td>$1 - 6 + 5 \neq 0$</td>
<td>$25 - 30 + 5 \neq 0$</td>
</tr>
<tr>
<td>$0 = 0$</td>
<td>$0 = 0$</td>
</tr>
</tbody>
</table>

Both numbers make the equation true. Both $-1$ and $-5$ are solutions.
How Can You Solve a Quadratic Equation by Factoring?

A quadratic equation can be solved by factoring the quadratic expression and then setting its factors equal to zero.

Find the solution to the quadratic equation \( x^2 - x - 12 = 0 \).

This quadratic equation can be solved by factoring because the quadratic expression \( x^2 - x - 12 \) can be written as the product of two factors, \( x - 4 \) and \( x + 3 \).

\[
    x^2 - x - 12 = (x - 4)(x + 3)
\]

To verify that this equation is true, use the FOIL method to multiply the two binomials.

\[
    (x - 4)(x + 3)
\]

First \( x \cdot x = x^2 \)
Outer \( x \cdot 3 = 3x \)
Inner \( -4 \cdot x = -4x \)
Last \( -4 \cdot 3 = -12 \)

FOIL \( x^2 + 3x - 4x - 12 \)

\[
    x^2 - x - 12
\]

Write the left side of the equation as a product of these two factors.

\( x^2 - x - 12 = 0 \)

\( (x - 4)(x + 3) = 0 \)

The product of two factors is 0 only if either of the factors is 0. Set each factor in the equation equal to 0 and solve for \( x \).

\[
    x - 4 = 0 \quad x + 3 = 0
\]

\[
    x = 4 \quad x = -3
\]

The solutions of the quadratic equation are 4 and -3.

Check both values of \( x \) to verify that the equation is true.

<table>
<thead>
<tr>
<th>Substitute ( x = 4 )</th>
<th>Substitute ( x = -3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 - x - 12 = 0 )</td>
<td>( x^2 - x - 12 = 0 )</td>
</tr>
<tr>
<td>( (4)^2 - 4 - 12 \not= 0 )</td>
<td>( (-3)^2 - (-3) - 12 \not= 0 )</td>
</tr>
<tr>
<td>( 16 - 4 - 12 \not= 0 )</td>
<td>( 9 + 3 - 12 \not= 0 )</td>
</tr>
<tr>
<td>( 0 = 0 )</td>
<td>( 0 = 0 )</td>
</tr>
</tbody>
</table>
In real-life problems modeled by quadratic equations, not all the solutions of the equation may make sense in the problem.

A rectangular garden is 1 foot longer than it is wide. Find the length and width of the garden if its area is 42 square feet.

- If the width is represented by \( w \), then the length can be represented by \( w + 1 \).
- Substitute these expressions into the formula for the area of a rectangle to model the problem situation with an equation.

\[
A = lw \\
42 = (w + 1)w
\]

- To find the width of the garden, solve this equation for \( w \).

First write the equation in standard quadratic form, \( ax^2 + bx + c = 0 \).

\[
w(w + 1) = 42 \\
w^2 + w = 42 \\
w^2 + w - 42 = 0
\]

- The quadratic expression \( w^2 + w - 42 \) can be factored.

\[w^2 + w - 42 = (w + 7)(w - 6)\]

Rewrite the equation with the quadratic expression factored.

\[(w + 7)(w - 6) = 0\]

- Set each factor equal to 0 and solve for \( w \).

\[
w + 7 = 0 \quad \quad w - 6 = 0
\]

\[
w = -7 \quad \quad w = 6
\]

- Width cannot be a negative value, so the solution \( w = -7 \) is not used. Use \( w = 6 \) as the solution to the equation.

The width of the garden is 6 feet. The length is \( w + 1 \), which is equal to 6 + 1, or 7 feet.
How Can You Solve a Quadratic Equation by Using the Quadratic Formula?

Another method used to solve quadratic equations is the quadratic formula. This method can be used to solve all quadratic equations.

The Quadratic Formula
The solutions to a quadratic equation in the standard form $ax^2 + bx + c = 0$ are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a$, $b$, and $c$ are the parameters of the quadratic equation.

Find the solutions to the equation $x^2 + x - 2 = 0$.

- The quadratic equation $x^2 + x - 2 = 0$ is written in standard form. Identify the values of the constants $a$, $b$, and $c$.

  $$a = 1$$
  $$b = 1$$
  $$c = -2$$

- Substitute these values for $a$, $b$, and $c$ in the quadratic formula, simplify the expression, and represent the two solutions separately.

  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  $$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2(1)}$$

  $$x = \frac{-1 \pm \sqrt{1 + 8}}{2}$$

  $$x = \frac{-1 \pm 3}{2}$$

  $x = \frac{-1 + 3}{2} = 1$ and $x = \frac{-1 - 3}{2} = -2$

The solutions to the equation are 1 and $-2$. 

When you find the square root of a number, remember to add the ± symbol in front of the square root symbol, $\sqrt{}$. For example,

$$x^2 = 9$$

$$\sqrt{x^2} = \pm \sqrt{9}$$

$x = 3$ shows that $(+3)^2 = 9$ and $(-3)^2 = 9$. 

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Find the solutions to the equation \(2x^2 + 4x - 3 = 0\).

- The quadratic equation \(2x^2 + 4x - 3 = 0\) is written in standard form. The values of the constants \(a\), \(b\), and \(c\) are its parameters.
  
  \[
  a = 2 \\
  b = 4 \\
  c = -3
  \]

- Substitute these values for \(a\), \(b\), and \(c\) in the quadratic formula, simplify the expression, and represent the two solutions separately.
  
  \[
  x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
  x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)} \\
  x = \frac{-4 \pm \sqrt{40}}{4} \\
  x = \frac{-4 + \sqrt{40}}{4} \text{ and } x = \frac{-4 - \sqrt{40}}{4}
  \]

- To approximate the roots of the equation, evaluate the final expressions using \(\sqrt{40} \approx 6.32\).
  
  \[
  x \approx \frac{-4 + 6.32}{4} \approx 0.58 \\
  x \approx \frac{-4 - 6.32}{4} \approx -2.58
  \]

The solutions to the equation are \(x \approx 0.58\) and \(x = -2.58\).
Try It
Estimate the roots of the equation $4x^2 + 1 = 8x$.

Write the quadratic equation in standard form.

$$\square \, x^2 - \square \, x + \square = 0$$

In the equation above,

$$a = \square, \quad b = \square, \quad \text{and} \quad c = \square.$$

Substitute the values of $a$, $b$, and $c$ into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-\square \pm \sqrt{\square^2 - 4 \cdot \square \cdot \square}}{2 \cdot \square}$$

$$x = \frac{\square \pm \sqrt{\square - \square}}{\square}$$

$$x = \frac{\square \pm \sqrt{\square}}{\square}$$

$$x = \frac{8 + \square}{\square} \approx \square \quad \text{and} \quad x = \frac{8 - \square}{\square} \approx \square$$

The approximate solutions of the equation are _______ and _______.

In the equation above, $a = 4$, $b = -8$, and $c = 1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 4 \cdot 1}}{2 \cdot 4}$$

$$x = \frac{8 \pm \sqrt{64 - 16}}{8}$$

$$x = \frac{8 \pm \sqrt{48}}{8}$$

$$x = \frac{8 + \sqrt{48}}{8} \approx 1.87 \quad \text{and} \quad x = \frac{8 - \sqrt{48}}{8} \approx 0.13$$

The approximate solutions of the equation are 1.87 and 0.13.
How Do You Apply the Laws of Exponents in Problem-Solving Situations?

When simplifying an expression with exponents, there are several rules, known as the laws of exponents, which must be followed.

- When multiplying terms with like bases, add the exponents.
  \[ x^a \cdot x^b = x^{(a+b)} \]
  Example:
  \[ x^4 \cdot x^2 = x^{(4+2)} = x^6 \]
  \[ (x \cdot x \cdot x \cdot x) \cdot (x \cdot x) = xxxxxx = x^6 \]

- When dividing terms with like bases, subtract the exponents.
  \[ \frac{x^a}{x^b} = x^{(a-b)} \]
  Example:
  \[ \frac{x^8}{x^3} = x^{(8-3)} = x^5 \]

Sometimes dividing variables with exponents produces negative exponents.

Example:
\[ \frac{x^3}{x^3} = x^{(3-3)} = x^{-2} \]
\[ \frac{xxx}{xxxxx} = \frac{1}{x^2} = x^{-2} \]

- A term with a negative exponent is equal to the reciprocal of that term with a positive exponent.
  \[ x^{-a} = \frac{1}{x^a} \]
  Example:
  \[ x^{-5} = \frac{1}{x^5} \]

- When raising a term with an exponent to a power, multiply the exponents.
  \[ (x^a)^b = x^{ab} \]
  Example:
  \[ (x^2)^7 = x^{2\cdot7} = x^{14} \]
  \[ (x^2)^7 = (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx) \]
  \[ (x^2)^7 = xxxxxxxxxxxxxx \]
  \[ (x^2)^7 = x^{14} \]

- Any base other than zero raised to the zero power equals one.
  \[ x^0 = 1 \]
  Example:
  \[ 8^0 = 1 \]
**Try It**

Simplify the expression $4x^3 \cdot -2x^4$.

\[
4x^3 \cdot -2x^4 = 4 \cdot x^3 \cdot -2 \cdot x^4 \\
= (4 \cdot -2) \cdot (x^3 \cdot x^4) \\
= -8 \cdot x^7 \\
= -8x^7
\]

---

**Try It**

Simplify the expression $(5a^3b^2)^2$.

\[
(5a^3b^2)^2 = 5^2 \cdot (a^3)^2 \cdot (b^2)^2 \\
= 25 \cdot a^6 \cdot b^4 \\
= 25a^6b^4
\]
Simplify the following expression: \( \left( \frac{x^7 y z^3}{x y^2 z} \right)^3 \).

Simplify the exponents in an expression raised to a power by multiplying the exponents.

Each term in the parentheses is raised to the power of three, so the rule must be applied to each of the variables.

\[
\left( \frac{x^7 y z^3}{x y^2 z} \right)^3 = \left( x^7 (y) (z^3) \right)^3 \cdot \frac{x^21 y^3 z^9}{x^3 y^{15} z^3}
\]

Next divide the like variables with exponents by subtracting the exponents. This rule can be used only if the bases are the same.

\[
x^{21} y^3 z^9 \div x^3 y^{15} z^3 = x^{(21 - 3)} \cdot y^{(3 - 15)} \cdot z^{(9 - 3)}
\]

Write the expression using only positive exponents.

\[
x^{18} y^{-12} z^6 = \frac{x^{18} z^6}{y^{12}}
\]

**Try It**

Find the area of a triangle with base \(3x^2 y^2\) and height \(2x^4 y^3\).

Substitute the given expressions for base and height into the formula for the area of a triangle, \(A = \frac{1}{2} bh\).

\[
A = \frac{1}{2} \cdot \left( \frac{3}{2} \cdot \frac{x^2}{y^2} \right) \cdot (x \square, x \square) \cdot (y \square, y \square)
\]

Simplify the expression.

\[
A = \frac{1}{2} \cdot \left( \frac{3 \cdot x^2 y^2}{2x^4 y^3} \right) \cdot (x \square + \square) \cdot (y \square + \square)
\]

\[
A = 3x \square y \square
\]

\[
A = \frac{1}{2} \cdot 3x^2 y^2 \cdot 2x^4 y^3
\]

\[
A = (\frac{1}{2} \cdot 3 \cdot 2) \cdot (x^2 \cdot x^4) \cdot (y^2 \cdot y^3)
\]

\[
A = \frac{1}{2} \cdot 6 \cdot (x^2 + 4) \cdot (y^2 + 3)
\]

\[
A = 3x^2 y^5
\]
Question 46
If the coefficient of $x^2$ in the function $y = -2x^2$ is multiplied by 2, how is the graph of $y = -2x^2$ affected?

A The graph is translated up 2 units.
B The graph is translated down 2 units.
C The graph becomes narrower.
D The graph becomes wider.

Question 47
How does the graph of the quadratic function $y = -\frac{1}{2}x^2$ compare to the graph of the quadratic function $y = \frac{1}{2}x^2$?

A The graph of $y = -\frac{1}{2}x^2$ is the graph of $y = \frac{1}{2}x^2$ reflected across the $y$-axis.
B The graph of $y = -\frac{1}{2}x^2$ is the graph of $y = \frac{1}{2}x^2$ reflected across the $x$-axis.
C The graph of $y = -\frac{1}{2}x^2$ is wider than the graph of $y = \frac{1}{2}x^2$.
D The graph of $y = -\frac{1}{2}x^2$ is narrower than the graph of $y = \frac{1}{2}x^2$.

Question 48
How does the graph of $y = x^2 - 1$ differ from the graph of $y = x^2 + 6$?

A The vertex of $y = x^2 - 1$ is 1 unit below the vertex of $y = x^2 + 6$.
B The vertex of $y = x^2 + 6$ is 6 units above the vertex of $y = x^2 - 1$.
C The vertex of $y = x^2 - 1$ is 5 units below the vertex of $y = x^2 + 6$.
D The vertex of $y = x^2 + 6$ is 7 units above the vertex of $y = x^2 - 1$. 
Question 49

A researcher collected data on profits resulting from different selling prices for a new mouthwash. She found that the curve that best fit the points on her scatterplot was the parabola shown below.

Which of the following is a correct interpretation of the researcher's graph?

A  As the selling price increases, the profits increase.
B  As the selling price increases, the profits decrease.
C  The selling price that results in the greatest profit is the highest possible selling price.
D  A selling price that is neither too high nor too low is best for profits.
Question 50

A stone is dropped from a height of 500 feet above the ground. The graph shows the stone’s distance from the ground at different times.

Which of the following is a correct interpretation of the graph?

A  The stone lands about 5 feet from where it was dropped.
B  The graph is a picture of the path of the stone as it falls.
C  The stone hits the ground between 5 seconds and 6 seconds after it is dropped.
D  The stone’s distance from the ground decreases at a constant rate until the stone hits the ground.
Question 51
The table below lists ordered pairs from a quadratic function.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>−3</td>
</tr>
<tr>
<td>2</td>
<td>−4</td>
</tr>
<tr>
<td>3</td>
<td>−3</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

What are the roots of the function?
A  −1 and 3
B  0 and 4
C  1 and 4
D  4 and −1

Question 52
Find the solution set to the quadratic equation below.
\[ 2x^2 + 3x = 9 \]

A  \{\frac{3}{2}, −3\}
B  \{2, −3\}
C  \{−2, 3\}
D  \{\frac{3}{2}, 3\}

Question 53
Which of the following quadratic equations has the solutions −1 and 5?
A  \(x^2 − 4x + 5 = 0\)
B  \(x^2 − 4x − 1 = 0\)
C  \(x^2 + 4x − 1 = 0\)
D  \(x^2 − 4x − 5 = 0\)
Objective 5

Question 54
For the quadratic equation $x^2 - 4x + 2 = 0$, the smaller root is between which 2 integers?

A 0 and 1
B −1 and 0
C 1 and 2
D −2 and −1

Question 55
The volume, $V$, of a sphere can be found using the following formula:

$$V = \frac{4}{3} \pi r^3$$

Which expression describes the volume of a sphere with radius $3x^2y$?

A $12x^6y^3\pi$
B $36x^6y^3\pi$
C $12x^5y^3\pi$
D $36x^6y^3\pi$
The student will demonstrate an understanding of geometric relationships and spatial reasoning.

For this objective you should be able to

- use transformational geometry to develop spatial sense; and
- use geometry to model and describe the physical world.

**How Do You Locate and Name Points on a Coordinate Plane?**

A coordinate grid is used to locate and name points on a plane. A coordinate grid is formed by two perpendicular number lines.

The x-axis and y-axis divide the coordinate plane into four regions, called quadrants. The quadrants are usually referred to by the Roman Numerals I, II, III, and IV.
Which of the points on the coordinate grid below satisfies the conditions $x > \frac{11}{2}$ and $y < -\frac{5}{2}$?

- Draw a vertical line through $x = \frac{11}{2}$. All the points to the right of this line have an $x$-coordinate greater than $\frac{11}{2}$.
- Draw a horizontal line through $y = -\frac{5}{2}$. All the points below this line have a $y$-coordinate less than $-\frac{5}{2}$.

Only point $B$, with the coordinates $(7, -3)$, satisfies both conditions.
**How Do You Find the Midpoint of a Line Segment?**

The midpoint of a line segment is the point that lies halfway between the segment's endpoints and divides it into two congruent parts. You can find the coordinates of the midpoint of a segment if you know the coordinates of its endpoints. Since the midpoint is halfway between the endpoints, its coordinates are the average of the coordinates of the endpoints.

Find the midpoint of \( \overline{AB} \).

To find the \( x \)-coordinate of the midpoint, find the \( x \)-value that is halfway between 2 and 6.

\[
(2 + 6) \div 2 = 4
\]

The \( x \)-coordinate of the midpoint is 4.

To find the \( y \)-coordinate of the midpoint, find the \( y \)-value that is halfway between 7 and 3.

\[
(7 + 3) \div 2 = 5
\]

The \( y \)-coordinate of the midpoint is 5.

The midpoint of \( \overline{AB} \) is (4, 5).
You can also find the midpoint of a line segment by using the following formula.

**Midpoint Formula**

For any two points \((x_1, y_1)\) and \((x_2, y_2)\), the coordinates of the midpoint of the line segment they determine are given by the formula below.

\[
M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]
Use the midpoint formula to find the midpoint, $M$, of $\overline{RS}$ with endpoints $R(3, 2)$ and $S(1, -4)$.

Replace the variables in the midpoint formula with values from the coordinates of the two given points.

$R(3, 2)$: $x_1 = 3$ and $y_1 = 2$

$S(1, -4)$: $x_2 = 1$ and $y_2 = -4$

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$M = \left( \frac{3 + 1}{2}, \frac{2 + (-4)}{2} \right)$$

The coordinates of the midpoint are the average of the $x$- and $y$-coordinates.

$$M = \left( \frac{4}{2}, \frac{-2}{2} \right)$$

$$M = (2, -1)$$

The midpoint of $\overline{RS}$ is the point $M(2, -1)$. Notice that point $M$ lies on the segment connecting point $R$ and point $S$ and divides the segment into two congruent parts.
How Can You Show Transformations on a Coordinate Plane?
Translations, reflections, and dilations can all be modeled on a coordinate plane. A figure has been translated or reflected if it has been moved without changing its shape or size. A figure has been dilated if its size has been changed proportionally.

Translations
A translation of a figure is a movement of the figure along a line. It can be described by stating how many units to the left or right the figure is moved and how many units up or down it is moved. A figure and its translated image are always congruent.

If \( \triangle ABC \) is translated 6 units to the right and 2 units up, what are the coordinates of the vertices of the translated triangle \( A'B'C' \)?

The vertices of \( \triangle ABC \) are \( A (0, 0) \), \( B (3, 1) \), and \( C (5, 4) \).
If the triangle is translated 6 units to the right, then 6 must be added to the \( x \)-coordinate of each vertex. If the triangle is translated 2 units up, then 2 must be added to the \( y \)-coordinate of each vertex.

The vertices of the translated figure are \( A' (6, 2) \), \( B' (9, 3) \), and \( C' (11, 6) \).
Reflections

A reflection of a figure is the mirror image of the figure across a line. The line is called the line of reflection. The new figure is a reflection of the original figure, with the line of reflection serving as the mirror. A figure and its reflected image are always congruent.

Each point of the reflected image is the same distance from the line of reflection as the corresponding point of the original figure, but on the opposite side of the line of reflection.

If the point \((-3, 4)\) is reflected across the \(x\)-axis, what will be the coordinates of its reflection?

- The \(x\)-coordinate of the point will be unchanged because the point is being reflected across the \(x\)-axis. The reflected point will have an \(x\)-coordinate of \(-3\).
- The \(y\)-coordinate of the point is 4 units above the \(x\)-axis, so the \(y\)-coordinate of the reflected point will be 4 units below the \(x\)-axis. The reflected point will have a \(y\)-coordinate of \(-4\).

The coordinates of the reflected point will be \((-3, -4)\). The point \((-3, 4)\) and its image \((-3, -4)\) are equally distant from the line of reflection, the \(x\)-axis.
If \( \triangle RST \) is reflected across the \( y \)-axis, what are the coordinates of its reflection, \( \triangle RS'T' \)?

- The vertices of \( \triangle RST \) are \( R (0, 2) \), \( S (3, 5) \), and \( T (1, 6) \).
- Point \( R \) is on the \( y \)-axis, so it is a common vertex for the triangles.
- Point \( S \) is 3 units to the right of the \( y \)-axis, so \( S' \) is 3 units to the left of the \( y \)-axis. The coordinates of \( S' \) are \((-3, 5)\).
- Point \( T \) is 1 unit to the right of the \( y \)-axis, so \( T' \) is 1 unit to the left of the \( y \)-axis. The coordinates of \( T' \) are \((-1, 6)\).

The vertices of \( \triangle RS'T' \) are \( R (0, 2) \), \( S' (-3, 5) \), and \( T' (-1, 6) \).

**Dilations**

A **dilation** is a proportional enlargement or reduction of a figure through a point called the center of dilation. The size of the enlargement or reduction is called the **scale factor** of the dilation.

- If the dilated image is larger than the original figure, then the scale factor \( > 1 \). This is called an **enlargement**.
- If the dilated image is smaller than the original figure, then the scale factor \( < 1 \). This is called a **reduction**.

A figure and its dilated image are always similar.
What scale factor was used to transform quadrilateral RSTV to quadrilateral R’S’T’V’?

To find the scale factor, compare the lengths of a pair of corresponding sides.

- Of the line segments that make up the quadrilaterals, the ones whose lengths are easiest to find are RS and R’S’, because they are horizontal.
- The length of RS is the difference between the x-coordinates of points R and S.
  \[ RS = 3 - 1 = 2 \]
- The length of R’S’ is the difference between the x-coordinates of points R’ and S’.
  \[ R’S’ = 5 - 2 = 3 \]
- You can tell from the graph that the figure has been enlarged, so the scale factor is greater than 1. Use this fact to be certain you state the ratio correctly.
- The scale factor is the ratio of their lengths.
  \[ \frac{R’S’}{RS} = \frac{3}{2} = 1.5 \]

The scale factor used to dilate quadrilateral RSTV to quadrilateral R’S’T’V’ is 1.5. Each side of the dilated quadrilateral is 1.5 times the length of the corresponding side of the original quadrilateral.
Another way to view a dilation is as a projection through a center of dilation.

Looking at a dilation in this way can help you see it as an enlargement or a reduction. $\triangle ABC$ has been dilated to form $\triangle RST$. 
\( \triangle LMN \) was dilated to form \( \triangle CDE \) with (0, 0) as the center of dilation. What scale factor was used to dilate \( \triangle LMN \)?

To find the scale factor, compare the lengths of a pair of corresponding sides of the two triangles.

- Use the lengths of \( \overline{CD} \) and \( \overline{LM} \) to find the scale factor, since both segments are horizontal.
- The length of \( \overline{CD} \) is the difference between the x-coordinates of points \( C \) and \( D \).
  \[ CD = 0 - (-2) = 2 \]
- The length of \( \overline{LM} \) is the difference between the x-coordinates of points \( L \) and \( M \).
  \[ LM = 0 - (-6) = 6 \]

- From the graph you can tell that \( \triangle LMN \) has been reduced, so the scale factor is less than 1. Use this fact to be certain you state the ratio correctly.
- The scale factor is the ratio of their lengths.
  \[ \frac{CD}{LM} = \frac{2}{6} = \frac{1}{3} \]

The scale factor used to dilate \( \triangle LMN \) to \( \triangle CDE \) is \( \frac{1}{3} \). Each side of the dilated triangle is \( \frac{1}{3} \) the length of the corresponding side of the original triangle.
Try It

ΔSTU has vertices S (0, −1), T (−1, 4), and U (3, 5). Find the coordinates of the vertices of its reflection across the x-axis.

Because this is a reflection across the x-axis, the _____-coordinates do not change.

The vertex S (0, −1) is 1 unit ____________ the x-axis.
S’ must be 1 unit ____________ the x-axis.
The coordinates of S’ are (_____ , _____).

The vertex T (−1, 4) is ____________ units above the x-axis.
T’ must be 4 units ____________ the x-axis.
The coordinates of T’ are (_____ , _____).

The vertex U (3, 5) is ____________ units above the x-axis.
U’ must be 5 units ____________ the x-axis.
The coordinates of U’ are (_____ , _____).

Because this is a reflection across the x-axis, the x-coordinates do not change. The vertex S (0, −1) is 1 unit below the x-axis. S’ must be 1 unit above the x-axis. The coordinates of S’ are (0, 1). The vertex T (−1, 4) is 4 units above the x-axis. T’ must be 4 units below the x-axis. The coordinates of T’ are (−1, −4). The vertex U (3, 5) is 5 units above the x-axis. U’ must be 5 units below the x-axis. The coordinates of U’ are (3, −5).

Now practice what you’ve learned.
Question 56

The triangle in the graph below will be dilated by a scale factor of 2, using the point (0, 0) as the center of dilation.

Which graph shows this dilation?
Objective 6

**Question 57**

Rectangle $R'S'T'V'$ is a dilation of rectangle $RSTV$. What is the scale factor of the dilation?

A $\frac{1}{3}$  
B $\frac{1}{2}$  
C 2  
D 3

**Question 58**

A triangle has side lengths 1, 1.5, and 2 units. Which of the following could be the lengths of the sides of a triangle that was formed by dilating the given triangle?

A 1, 3, and 4 units  
B 2, 4, and 6 units  
C 4, 4.5, and 5 units  
D 4, 6, and 8 units

**Question 59**

Which of the following sets of points would be three of the vertices of the pentagon below reflected across the $y$-axis?

A $(-5, 6), (-4, 1), (-3, 5)$  
B $(5, -6), (4, -1), (3, -5)$  
C $(6, 5), (1, 4), (5, 3)$  
D $(-5, -6), (-4, -1), (-3, -5)$

**Question 60**

The quadrilateral with vertices $R(1, 1)$, $S(1, 4)$, $T(5, 8)$, and $V(7, -2)$ is translated 2 units to the right and 3 units down. Which of the following are the coordinates of two of the vertices of the translated quadrilateral?

A $(3, 4), (7, 5)$  
B $(-1, -2), (3, 5)$  
C $(7, 11), (3, 4)$  
D $(3, -2), (7, 5)$
**Question 61**

Triangle $LMN$ is dilated by a scale factor of 2 with $(0, 0)$ as the center of dilation. What will be the coordinates of $L'$, $M'$, and $N'$ of the dilated triangle?

- **A** $L'$ (5, 7), $M'$ (5, 10), and $N'$ (11, 7)
- **B** $L'$ (3, 8), $M'$ (3, 12), and $N'$ (6, 6)
- **C** $L'$ (6, 8), $M'$ (6, 12), and $N'$ (12, 6)
- **D** $L'$ (5, 6), $M'$ (5, 8), and $N'$ (8, 6)

**Question 62**

The endpoints of $RS$ are $R (-3, -7)$ and $S (3, -5)$. What are the coordinates of the midpoint of $RS$?

- **A** (6, 2)
- **B** (6, 12)
- **C** (0, -12)
- **D** (0, -6)
Objective 6

**Question 63**

For which of the points on the graph is $\frac{-7}{2} < x < \frac{3}{2}$?

- A  Point R
- B  Point S
- C  Point T
- D  Point U

**Question 64**

The coordinates of a given point are $(m, 2n)$. What are the coordinates of the point when it is translated 2 units to the right?

- A  $(m + 2, 2n + 2)$
- B  $(m, 2n + 2)$
- C  $(2m, 4n)$
- D  $(m + 2, 2n)$

**Question 65**

Which point best represents $\left(\frac{8}{3}, -\frac{9}{5}\right)$?

- A  Point L
- B  Point M
- C  Point N
- D  Point P
For this objective you should be able to use geometry to model and describe the physical world.

**How Do You Recognize a Solid from Different Perspectives?**

Given a drawing of a three-dimensional figure, a solid, you should be able to recognize other drawings that represent the same figure from a different perspective.

A three-dimensional figure can be represented by drawing the figure from three different views: front, top, and side.

To recognize the solid from different perspectives, you must visualize what the solid would look like if you were seeing it from the front, from above, or from one side.

The solid below is made up of many small cubes. Can you visualize what it would look like from the front, from above, and from a side?

These are the front, top, and side views of this solid.
What Kinds of Problems Can You Solve with Geometry?

The laws of geometry govern the physical world around us. You can solve many types of problems using geometry, including problems involving these geometric concepts:

- the area or perimeter of figures;
- the measures of the sides or angles of polygons;
- the surface area and volume of solid figures;
- the ratios of the sides of similar figures; and
- the relationship between the sides of a right triangle.

Joan is making a tablecloth with a lace border. The tablecloth fabric costs $4.25 per square yard. The lace border costs $0.35 per foot. What is the approximate total cost of the fabric and lace border for a rectangular tablecloth that is 3 feet wide and 5 feet long?

To answer this question, you need to know the area of the tablecloth to find the cost of the fabric, and you need to know its perimeter to find the cost of the lace border.

- Use the formula for the area of a rectangle. The dimensions are given in feet.

\[ A = lw \]
\[ A = 5 \times 3 = 15 \text{ ft}^2 \]

The area of the tablecloth is 15 square feet. You want to know the area in square yards.

\[ 15 \text{ ft}^2 \div 9 \text{ ft}^2 \text{ per yd}^2 \approx 1.67 \text{ yd}^2 \]

Joan needs about 1.67 square yards of fabric.

- Fabric costs $4.25 per square yard.

\[ 1.67 \text{ yd}^2 \times $4.25 \text{ per yd}^2 \approx $7.10 \text{ for fabric} \]

- Use the perimeter formula to find the perimeter of the tablecloth.

\[ P = 2(l + w) \]
\[ P = 2(5 + 3) = 16 \text{ ft} \]

The perimeter of the tablecloth is 16 feet.

- Lace border costs $0.35 per foot.

\[ 16 \text{ ft} \times $0.35 \text{ per ft} = $5.60 \text{ for lace border} \]

The approximate cost of these materials is $7.10 + $5.60 = $12.70.
A jewelry designer is working with a design that is a regular pentagon inscribed in a circle. He needs to cut a small stone to fit into each of the triangles shown below. What is the measure of the angle indicated?

- Think about what you know.
  
  The angles at the center of a pentagon have a sum of 360°, and they are congruent because the pentagon is a regular 5-sided polygon.

- Use what you know to write an equation. Let \( x \) represent the measure of the angle.
  
  \[
  x = \frac{360}{5} 
  \]

  \[
  x = 72 
  \]

  The measure of the angle indicated is 72°.

An architect is constructing a scale model of a building with a rectangular base. The actual building will be 300 feet tall, but the scale model is 18 inches tall. The dimensions of the building’s base are 200 feet by 150 feet. What are the dimensions of the base of the scale model?

Use a proportion to find each of the dimensions of the base of the scale model.

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{18}{300} = \frac{l}{200} )</td>
<td>( \frac{18}{300} = \frac{w}{150} )</td>
</tr>
<tr>
<td>300( l ) = 18 \cdot 200</td>
<td>300( w ) = 18 \cdot 150</td>
</tr>
<tr>
<td>300( l ) = 3600</td>
<td>300( w ) = 2700</td>
</tr>
<tr>
<td>( l ) = 12 in.</td>
<td>( w ) = 9 in.</td>
</tr>
</tbody>
</table>

The dimensions of the model’s base are 12 inches by 9 inches.
Try It

The dining area of a restaurant includes a patio 25 feet wide by 40 feet long. On an architect's scale drawing of the restaurant, the width of the patio is 5 inches. What is the length of the patio in the scale drawing?

Use a proportion to find the length of the patio in the scale drawing. Find the ratios of corresponding measurements.

Width: 5
Length: l

Write a proportion and solve it.

\[
\frac{5}{25} = \frac{l}{40}
\]

\[\frac{5}{25} \cdot l = 5 \cdot 40\]

\[25l = 200\]

\[l = 8\]

The length of the patio in the scale drawing is 8 inches.
What Is the Pythagorean Theorem?

The Pythagorean Theorem is a relationship among the lengths of the sides of a right triangle. This special relationship applies only to right triangles.

The sides of a right triangle have special names.

- The hypotenuse of a right triangle is the longest side of the triangle. The hypotenuse is always opposite the right angle in the triangle. In the diagram below, the length of the hypotenuse is represented by $c$.

- The legs of the right triangle are the two sides that form the right angle. In the diagram below, the lengths of the legs are represented by $a$ and $b$.

The Pythagorean Theorem can be stated algebraically or verbally.

**Algebraic**

In any right triangle with legs $a$ and $b$ and hypotenuse $c$, $a^2 + b^2 = c^2$.

**Verbal**

In any right triangle the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

The Pythagorean Theorem can also be interpreted with a geometric model.

**Geometric**

In any right triangle with legs $a$ and $b$, the squares of the lengths of the legs ($a^2$ and $b^2$) are equal to the square of the length of the hypotenuse ($c^2$).
Does the model below demonstrate the Pythagorean Theorem?

A triangle is a right triangle if its sides satisfy the Pythagorean Theorem. A geometric model of the Pythagorean Theorem uses squares, not rectangles, to show that the sum of the areas formed by the legs is equal to the area formed by the hypotenuse. The model above does not demonstrate the Pythagorean Theorem.

Look at another model.

The square formed by the hypotenuse is 10 \cdot 10 units. It has an area of $10 \cdot 10 = 100$ square units.

The square formed by one leg is 6 \cdot 6 units. It has an area of $6 \cdot 6 = 36$ square units.

The square formed by the other leg is 8 \cdot 8 units. It has an area of $8 \cdot 8 = 64$ square units.

Since $100 = 36 + 64$, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs.

The second model does show that the sides of the triangle satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$. The sides form a right triangle.
Try It
The model below shows how three squares could be joined to form a triangle. Do the squares form a right triangle?

The longest side is _____ units long, so it is the hypotenuse.
The legs are _____ units and _____ units long.
The square formed by the hypotenuse is _____ by _____ units.
It has an area of _____ • _____ = _____ square units.
The square formed by the shorter leg is _____ by _____ units.
It has an area of _____ • _____ = _____ square units.
The square formed by the longer leg is _____ by _____ units.
It has an area of _____ • _____ = _____ square units.
Since _____ = _____ + _____, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs.
Yes, the squares form a right triangle.

The longest side is 13 units long, so it is the hypotenuse. The legs are 5 units and 12 units long. The square formed by the hypotenuse is 13 by 13 units. It has an area of 13 • 13 = 169 square units. The square formed by the shorter leg is 5 by 5 units. It has an area of 5 • 5 = 25 square units. The square formed by the longer leg is 12 by 12 units. It has an area of 12 • 12 = 144 square units. Since 169 = 25 + 144, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs. Yes, the squares form a right triangle.

Now practice what you've learned.
Question 66

The object below is built from blocks.

Which is not a top, front, or side view of the object?

A

B

C

D

Answer Key: page 236
Question 67
Look at the drawing of the solid below.

Which represents the top view of this solid?

A  
B  
C  
D  

Question 68
An interior decorator painted two rectangular panels. One panel is 10 feet by 20 feet, and the other is 4 feet by 15 feet. The can of paint she used covers at most 400 square feet. She then used all the paint that remained in the can to completely paint a third rectangular panel. Which of the following is a reasonable estimate of the dimensions of the third panel?

A  12 ft by 20 ft  
B  15 ft by 15 ft  
C  10 ft by 16 ft  
D  10 ft by 12 ft
**Question 69**

Each set of squares below can be joined at their vertices to form a triangle. Which set of squares could not be used to form the sides of a right triangle?

A

![Diagram of squares A](image)

B

![Diagram of squares B](image)

C

![Diagram of squares C](image)

D

![Diagram of squares D](image)

**Question 70**

An archaeologist is making a scale drawing of the foundation of an ancient building. The foundation is a rectangle that measures 18 feet by 45 feet. If the shorter dimension of the drawing is 4 inches, what is the longer dimension in inches?

Record your answer and fill in the bubbles. Be sure to use the correct place value.
**Question 71**

Ed is installing a new bathroom sink countertop. The rectangular countertop is 5 feet 4 inches long by 2 feet 2 inches wide. He plans to tile the countertop with square tiles that are 2 inches on each side. The circular sink has a diameter of 16 inches.

What is the minimum number of tiles Ed will need to cover the countertop area, not including the sink?

A 732  
B 366  
C 410  
D 404

**Question 72**

Which model below uses the Pythagorean Theorem to show that the triangle is a right triangle?

A  
B  
C  
D  

Answer Key: page 237
Objective 8

The student will demonstrate an understanding of the concepts and uses of measurement and similarity.

For this objective you should be able to

- use procedures to determine measures of solids;
- use indirect measurement to solve problems; and
- describe how changes in dimensions affect linear, area, and volume measurements.

How Do You Find the Surface Area of Solids?

You can use models or formulas to find the surface area of prisms, cylinders, and other solids.

- A prism is a solid figure with two bases. The bases are congruent polygons. The other faces of the prism are rectangles. The prism is named by the shape of its bases. For example, a triangular prism has two triangles as its bases.

  ![Triangular Prism](image1)

- A cylinder is a solid figure with two congruent circular bases and a curved surface.

  ![Cylinder](image2)
Like the area of a plane figure, the surface area of a solid figure is measured in square units.

- The **total surface area** of a solid figure is equal to the sum of the areas of all its surfaces.
- The **lateral surface area** of a solid figure is equal to the sum of the areas of all its faces and curved surfaces. It does not include the area of the figure’s bases.

One way to find the surface area of a solid figure is to use a net of the figure. A net of a 3-dimensional figure is a 2-dimensional drawing that shows what the figure would look like when opened up and unfolded with all its surfaces laid out flat. Use the net to find the area of each surface.

You can also find the surface area of a solid figure by using a formula. Substitute the appropriate dimensions of the figure into the formula and calculate its surface area. The formulas for total surface area and lateral surface area of several solid figures are included in the Mathematics Chart.
Find the lateral surface area and the total surface area of the cylinder shown below to the nearest tenth of a square centimeter.

Use the formula for the lateral surface area of a cylinder, $S = 2\pi rh$.

- The diameter of the cylinder shown on the diagram is 3.2 centimeters. The radius, $r$, is $\frac{1}{2}$ the diameter. Since $\frac{1}{2} \times 3.2 = 1.6$, the radius of the cylinder is 1.6 cm.
- The height, $h$, shown on the diagram is 6.4 centimeters.
- Substitute the values of $r$ and $h$ into the formula.

$$S = 2\pi rh$$

$$S = 2(\pi)(1.6)(6.4)$$

$S = 64.3398$

The lateral surface area is approximately 64.3 cm$^2$.

The formula for the total surface area of a cylinder is $S = 2\pi rh + 2\pi r^2$.

- The area of the lateral surface, $2\pi rh \approx 64.3398$ cm$^2$, was found above.
- Find the area of the two circular bases.
  Substitue the value of the radius, $r = 1.6$, into the second part of the formula, $2\pi r^2$.

$$2\pi r^2 = 2(\pi)(1.6)^2 \approx 16.0850 \text{ cm}^2$$

- To calculate the total surface area of the cylinder, add the lateral surface area and the area of the two bases.

$$S = 64.3398 + 16.0850 \approx 80.4248$$

The total surface area of the cylinder is approximately 80.4 cm$^2$. 
A storage shed has rectangular sides and a roof in the shape of a half-cylinder. Both the sides and the roof of the shed are to be painted. The dimensions of the shed are shown below.

Find the total area of the surfaces to be painted to the nearest square foot.

- Calculate the area of the rectangular sides of the storage shed. There are two sides with dimensions 6 ft by 8.5 ft. Each of these sides has an area of $6 \times 8.5 = 51 \text{ ft}^2$. There are two sides with dimensions 5 ft by 8.5 ft. Each of these sides has an area of $5 \times 8.5 = 42.5 \text{ ft}^2$. The four sides have a total area of $2(51) + 2(42.5) = 187 \text{ ft}^2$.

- Calculate the surface area of the roof. The roof is a half-cylinder with radius 3 ft and height 5 ft. The formula for the surface area of a cylinder is $S = 2\pi rh + 2\pi r^2$. Substitute the values for $r$ and $h$ into the formula.

$$S = 2\pi rh + 2\pi r^2$$
$$S = 2\pi \cdot 3 \cdot 5 + 2\pi \cdot 3^2$$
$$S = 30\pi + 18\pi$$
$$S = 48\pi$$
$$S \approx 150.80 \text{ ft}^2$$

The area of the cylinder is about 150.80 $\text{ft}^2$. Since the roof is only half a cylinder, divide the surface area by 2. The area of the roof is approximately 75.40 $\text{ft}^2$.

- Add the area of the roof to the total area of the sides of the building to find the total area to be painted.

$$187 \text{ ft}^2 + 75.40 \text{ ft}^2 \approx 262.40 \text{ ft}^2$$

To the nearest square foot, the area to be painted is 262 $\text{ft}^2$. 

Try It

The net of a triangular prism is shown on the coordinate grid below.

The height of each triangle (indicated by the dotted lines) is approximately 1.7 units. Use the grid to find the other dimensions of the prism and its total surface area to the nearest square unit.

The surface area of the prism is equal to the _______________ of the areas of all its faces.

The total surface area of the triangular prism is _______ square units.

Find the area of a triangular face.  
\[ A = \frac{1}{2} \cdot bh \]
\[ A = \frac{1}{2} \cdot _____ \cdot _____ \]
\[ A = _____ \]
Each triangular face has an area of _____ square units.
The prism has _____ triangular faces.
Since 2 \cdot _____ = _____, their combined area is _____ square units.

Find the area of a rectangular face.  
\[ A = lw \]
\[ A = _____ \cdot _____ \]
\[ A = _____ \]
Each rectangular face has an area of _____ square units.
The prism has _____ rectangular faces.
Since 3 \cdot _____ = _____, their combined area is _____ square units.

The total surface area of the triangular prism is \[ _____ + _____ = _____ \] square units.

The surface area of the prism is equal to the **sum** of the areas of all its faces.
\[ A = \frac{1}{2} \cdot bh = \frac{1}{2} \cdot 2 \cdot 1.7 = 1.7 \]
Each triangular face has an area of 1.7 square units. The prism has 2 triangular faces. Since 2 \cdot 1.7 = 3.4, their combined area is 3.4 square units.

\[ A = lw = 8 \cdot 2 = 16 \]
Each rectangular face has an area of 16 square units. The prism has 3 rectangular faces. Since 3 \cdot 16 = 48, their combined area is 48 square units.

The total surface area of the triangular prism is \[ 3.4 + 48 = 51.4 \] square units.
What Is the Volume of a Solid?
The volume of a solid is a measure of the space it occupies. Volume is measured in cubic units.

You can use formulas or models to find the volume of solid figures. The formulas for calculating the volume of several solid figures are in the Mathematics Chart.

When using a formula to find the volume of a solid, follow these guidelines:

- Identify the solid figure you are working with. This will help you select the correct volume formula.
- Use models to help visualize the solid and to assign the variables in the volume formula. A model can also be used to find the dimensions of a figure.
- Substitute the appropriate dimensions of the figure for the corresponding variables in the volume formula.
- Calculate the volume. State your answer in cubic units.
The net of a rectangular prism is shown below. Use the ruler provided on the Mathematics Chart to measure the dimensions of the figure to the nearest tenth of a centimeter. Use these dimensions to find the volume of the rectangular prism.

Use the formula for the volume of a prism, \( V = Bh \), in which \( B \) represents the area of the base of the prism and \( h \) represents the prism's height.

- Find the area of the base, \( B \). The base is a rectangle, so its area is equal to its length times its width.
  
  Measure the length and width using the centimeter ruler. The length is 5.0 cm, and the width is 3.5 cm. Calculate the area of the base using \( A = lw \).

\[
B = 5.0 \cdot 3.5
\]

\[
B = 17.5 \text{ cm}^2
\]

- Measure the height of the prism: \( h = 2.0 \text{ cm} \).

- Substitute the area of the base, \( B \), and the height, \( h \), into the formula for the volume of a prism, \( V = Bh \).

\[
V = 17.5 \cdot 2.0
\]

\[
V = 35 \text{ cm}^3
\]

The prism has a volume of 35 cubic centimeters.
**Try It**

Which solid has the greater volume: a cylinder 3 inches high with a radius of 2 inches or a cone of the same radius that is 8.5 inches high?

**Cylinder**

The formula for the volume of a cylinder is \( V = Bh \). Find \( B \), the area of the base of the cylinder.

The base of the cylinder is a ____________.

Use the formula \( A = \pi r^2 \).

Substitute _____ for \( r \).

\[
A = \pi r^2
\]

\[
A = \pi \cdot _____
\]

\[
A \approx _____
\]

\[
B \approx _____ \text{ in}^2
\]

Substitute 12.57 for \( B \) and _____ for \( h \) in the formula for the volume of a cylinder.

\[
V = Bh
\]

\[
V \approx 12.57 \cdot _____
\]

\[
V \approx _____
\]

The volume of the cylinder is about _____ cubic inches.

The ____________ has the greater volume.

**Cone**

The formula for the volume of a cone is \( V = \frac{1}{3} Bh \). Find \( B \), the area of the ____________ of the cone.

The base of the cone is a ____________.

Use the formula \( A = \pi r^2 \).

Substitute _____ for \( r \).

\[
A = \pi r^2
\]

\[
A = \pi \cdot _____
\]

\[
A \approx _____
\]

\[
B \approx _____ \text{ in}^2
\]

Substitute 12.57 for \( B \) and _____ for \( h \) in the formula for the volume of a cone.

\[
V = \frac{1}{3} Bh
\]

\[
V \approx \frac{1}{3} \cdot 12.57 \cdot _____
\]

\[
V \approx _____
\]

The volume of the cone is about _____ cubic inches.
**Objective 8**

How Can You Solve Problems Using the Pythagorean Theorem?

The Pythagorean Theorem is a relationship among the lengths of the three sides of a right triangle. The Pythagorean Theorem applies only to right triangles.

- In any right triangle with leg lengths \( a \) and \( b \) and hypotenuse length \( c \), \( a^2 + b^2 = c^2 \).
- If the side lengths of any triangle satisfy the equation \( a^2 + b^2 = c^2 \), then the triangle is a right triangle, and \( c \) is its hypotenuse.

Any set of three whole numbers that satisfy the Pythagorean Theorem is called a Pythagorean triple. The set of numbers \( \{5, 12, 13\} \) forms a Pythagorean triple because these numbers satisfy the Pythagorean Theorem. To show this, substitute 13 for \( c \) in the formula—since 13 is the greatest number—and substitute 5 and 12 for \( a \) and \( b \).

\[
\begin{align*}
5^2 + 12^2 &= 13^2 \\
25 + 144 &= 169 \\
169 &= 169
\end{align*}
\]

A triangle with side lengths 5, 12, and 13 units is a right triangle.

Any multiple of a Pythagorean triple is also a Pythagorean triple. Since the set of numbers \( \{5, 12, 13\} \) is a Pythagorean triple, the triple formed by multiplying each number in the set by 2, \( \{10, 24, 26\} \), is also a Pythagorean triple. A triangle with side lengths 10, 24, and 26 units is also a right triangle.
Would a triangle with side lengths 3 inches, 4 inches, and 5 inches form a right triangle?

Determine whether the side lengths satisfy the Pythagorean Theorem. Since 5 is the greatest length, it would be the length of the triangle’s hypotenuse. Substitute 5 for \( c \) in the formula. Substitute 3 and 4 for \( a \) and \( b \), the two legs.

\[
\begin{align*}
3^2 + 4^2 & \equiv 5^2 \\
9 + 16 & \equiv 25 \\
25 & \equiv 25
\end{align*}
\]

Since the side lengths satisfy the Pythagorean Theorem, the triangle is a right triangle.

This example also shows that the set \{3, 4, 5\} forms a Pythagorean triple.

A right triangle has a side length of 24 meters and a hypotenuse of 25 meters. Find the length of the third side.

Substitute 25, the hypotenuse, for \( c \) and 24, the length of one side, for \( b \).

\[
\begin{align*}
a^2 + b^2 &= c^2 \\
a^2 + 24^2 &= 25^2 \\
a^2 + 576 &= 625 \\
-576 &= -576 \\
a^2 &= 49 \\
a &= \sqrt{49} \\
a &= 7
\end{align*}
\]

The length of the third side is 7 meters.
On a tour around a lake, visitors ride a tour bus 3 miles south and 5 miles east. Then they ride a boat across the lake back to the starting point. Their journey forms a right triangle.

Approximately how many miles is the trip across the lake?

The bus route and the boat route form a right triangle. The two parts of the journey traveled by bus form the legs of the right triangle. Their lengths are given as 3 miles and 5 miles. The boat’s return path across the lake forms the hypotenuse, \( c \).

Use the Pythagorean Theorem to find the length of the boat’s path across the lake.

- Substitute 3 and 5 for the legs, \( a \) and \( b \).
  \[
  a^2 + b^2 = c^2 \\
  3^2 + 5^2 = c^2 \\
  9 + 25 = c^2 \\
  34 = c^2 \\
  \sqrt{34} = c
  \]

- Find a decimal approximation of \( \sqrt{34} \).
  \[
  \sqrt{34} = 5.831
  \]

Rounded to the nearest tenth, the trip across the lake is approximately 5.8 miles.
How Can You Use Proportional Relationships to Solve Problems?

You can use proportional relationships to find missing side lengths in similar figures. To solve problems that involve similar figures, follow these guidelines:

- Identify which figures are similar and which sides correspond. Similar figures have the same shape, but not necessarily the same size. The lengths of the corresponding sides of similar figures are proportional.

Triangle $ABC$ is similar to triangle $RST$.

\[ \triangle ABC \sim \triangle RST \]

\[ \frac{AB}{RS} = \frac{BC}{ST} = \frac{AC}{RT} \]

- Write a proportion and solve it.
- Answer the question asked.

The triangles in the drawing below are similar.

Find the length of $\overline{AC}$.

- Write a proportion comparing the ratio of the unknown side and its corresponding side to the ratio of a pair of corresponding sides whose lengths are known.

$\overline{AC}$ corresponds to $\overline{DF}$. The length of $\overline{AC}$ is unknown; $DF = 7.5$ ft. $\overline{BC}$ and $\overline{EF}$ are a pair of corresponding sides with known lengths; $BC = 4$ ft, and $EF = 10$ ft.

\[ \frac{BC}{EF} = \frac{AC}{DF} \]

- Substitute the known values.

\[ \frac{4}{10} = \frac{AC}{7.5} \]

- Use cross products to solve.

\[ 4(7.5) = 10 \cdot AC \]

\[ 30 = 10 \cdot AC \]

\[ 3 = AC \]

The length of $\overline{AC}$ is 3 feet.
Try It

Two regular hexagons are inscribed in circles. The smaller hexagon has a side length of 12 centimeters, and the larger one has a side length of 60 centimeters. If the radius of the larger circle is 52 centimeters, what is the radius of the smaller circle?

Since the hexagons are similar figures, the ratios of the lengths of their corresponding sides will be proportional.

\[
\frac{\text{large}}{\text{small}} = \frac{12}{r}
\]

Use cross products to solve.

\[
12 \cdot r = 52 \cdot 12
\]

\[
r = \frac{52 \cdot 12}{12} = 52 \cdot 10.4
\]

The radius of the smaller circle is 10.4 centimeters.

How Is the Perimeter of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The perimeter of the dilated figure will change by the same scale factor.

To dilate a figure means to enlarge or reduce it by a given scale factor. For example, the quadrilateral on the left has been dilated (enlarged) by a scale factor of 2.5 to form the quadrilateral on the right.

The perimeter of the larger quadrilateral is 2.5 times the perimeter of the smaller quadrilateral.

If the dimensions of two similar figures are in the ratio \( \frac{a}{b} \), then their perimeters will be in the ratio \( \frac{a}{b} \).
In the drawing below, rectangle B is a dilation of rectangle A by a scale factor of 1.5.

What effect should this dilation have on the perimeter of rectangle B? The perimeter of rectangle B should increase by the same factor, 1.5.

- To prove this, first find the perimeter of rectangle A.
  \[ P = 2(l + w) = 2(5 + 3) = 2(8) = 16 \text{ units} \]

- Use the scale factor to find the perimeter of rectangle B.

  \[ 16 \cdot 1.5 = 24 \]

- Use the formula to find the perimeter of rectangle B.
  
  The dimensions of rectangle B equal the dimensions of rectangle A multiplied by the scale factor, 1.5.

  Find the length and width of rectangle B.
  \[ l = 5 \cdot 1.5 = 7.5 \]
  \[ w = 3 \cdot 1.5 = 4.5 \]

  Find the perimeter.
  \[ P = 2(l + w) \]
  \[ P = 2(7.5 + 4.5) \]
  \[ P = 2(12) \]
  \[ P = 24 \text{ units} \]

- Compare the two perimeters.
  \[ \frac{24}{16} = \frac{1.5}{1} \]

The perimeter of rectangle B increased by a factor of 1.5, the same factor by which its dimensions increased.
How Is the Area of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The area of the dilated figure will change by the square of the scale factor.

If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their areas will be in the ratio $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$.
A 5-by-7-inch photograph is enlarged by a scale factor of 4. How is the area of the photograph affected?

The area of the photograph should increase by the square of the scale factor: \(4^2 = 16\). The area should increase by a factor of 16.

- To prove this, first find the area of the original photograph.

\[
A = lw \\
A = 7 \cdot 5 \\
A = 35 \text{ in.}^2
\]

- Use the scale factor to find the area of the enlarged photograph.

\[
35 \cdot (4^2) = 35 \cdot 16 = 560
\]

- Use the formula to find the area of the enlarged photo. The dimensions of the enlarged photo are the dimensions of the original photo multiplied by the scale factor, 4.

Find the length and width of the enlarged photo.

\[
l = 7 \cdot 4 = 28 \\
w = 5 \cdot 4 = 20
\]

Find the area.

\[
A = lw \\
A = 28 \cdot 20 \\
A = 560 \text{ in.}^2
\]

- The ratio of the two areas is \(\frac{560}{35}\). When reduced, this ratio is \(\frac{16}{1}\), or \((\frac{4}{1})^2\).

The area of the enlarged photograph increased by a factor of 16.

**Try It**

A certain bakery sells two sizes of round cakes. The radius of the small cake is \(\frac{1}{2}\) the radius of the large cake. If the top of the large cake has an area of 200 in.\(^2\), what is the area of the top of the small cake?
How Is the Volume of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The volume of the dilated figure will change by the cube of the scale factor.

If the dimensions of two similar solid figures are in the ratio \( \frac{a}{b} \), then their volumes will be in the ratio \( \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3} \).
A rectangular prism with a volume of 60 cubic units is dilated by a scale factor of 3. What is the volume of the dilated prism?

- The volume of the original prism is 60 cubic units.
- Find the volume of the dilated prism.

\[ 60 \cdot (3^3) = 60 \cdot 27 = 1620 \]

The volume of the dilated prism is 1620 cubic units.

**Try It**

A breakfast-cereal manufacturer is using a scale factor of 2.5 to increase the size of one of its cereal boxes. If the volume of the original cereal box was 240 in.\(^3\), what is the volume of the enlarged box?

If the dimensions of the box are increased by a scale factor of \(2.5\), then the volume of the box will increase by the \(\text{cube}\) of the scale factor.

\[ V = \text{original volume} \cdot (2.5)^3 \]

\[ V = 240 \cdot (2.5)^3 \]

\[ V = 240 \cdot 15.625 \]

\[ V = 3750 \text{ in.}^3 \]

The volume of the enlarged box is 3750 cubic inches.

Now practice what you've learned.
Question 73
The net below can be folded to form a cylinder.

What is the approximate total surface area of the cylinder?

A  64 square meters  
B  83 square meters  
C  246 square meters  
D  286 square meters

Answer Key: page 237

Question 74
The net of a triangular prism is shown below. Use the ruler on the Mathematics Chart to measure the dimensions of the prism to the nearest tenth of a centimeter.

Which is closest to the total surface area of the prism?

A  37 cm²  
B  23 cm²  
C  47 cm²  
D  40 cm²

Answer Key: page 237
Question 75
Use the ruler on the Mathematics Chart to measure the dimensions of the figure to the nearest tenth of a centimeter.

Which equation could be used to find the volume of the figure in cubic centimeters?

A \[ V = \frac{1}{3}\pi(3.1)^2(4) + \frac{1}{3}\pi(3.1)^2(7.6) \]
B \[ V = \frac{1}{3}\pi(3.1)(7.6)^2 + \pi(3.1)(4)^2 \]
C \[ V = \frac{1}{3}\pi(3.1)^2(7.6) + \pi(3.1)^2(4) \]
D \[ V = \pi(3.1)(7.6)^2 + \pi(3.1)(4)^2 \]

Question 76
A pipe in the shape of a cylinder with a 30-inch diameter is to go through a passageway shaped like a rectangular prism. The passageway is 3 ft high, 4 ft wide, and 6 ft long. The space around the pipe is to be filled with insulating material.

What is the volume, to the nearest cubic foot, of the space to be filled with insulating material?

A 72 ft³
B 43 ft³
C 30 ft³
D 29 ft³
Question 77
Jillian walks from the parking-lot entrance of a park to the scenic overlook by following a sidewalk along the edge of the park. She walks back to the parking lot by taking a shortcut through the park. The drawing below shows her journey.

![Diagram showing Jillian's journey](image)

To the nearest foot, how much shorter was her trip back to the parking lot than her walk to the scenic overlook?

A 417 ft  
B 213 ft  
C 562 ft  
D 261 ft

**Answer Key: page 238**

Question 78
\(\triangle LMN\) and \(\triangle RST\) are similar.

![Diagram showing similar triangles](image)

What is the length of \(ST\)?

A 6.4 units  
B 7.75 units  
C 13.3 units  
D 8.75 units

**Answer Key: page 238**

Question 79
An art store sells two sizes of rectangular poster boards. The smaller poster board has a width of 18 inches and a height of 24 inches. The larger poster board is similar to the smaller one and has a height of 28 inches. What is the width in inches of the larger poster board?

Record your answer and fill in the bubbles. Be sure to use the correct place value.

**Answer Key: page 238**

Question 80
The ratio of the diameter of a larger circle to the diameter of a smaller circle is \(\frac{3}{2}\). Which number represents the ratio of the area of the larger circle to the area of the smaller circle?

A \(\frac{3}{2}\)  
B \(\frac{2}{3}\)  
C \(\frac{4}{9}\)  
D \(\frac{9}{4}\)

**Answer Key: page 239**
**Question 81**
Triangle $MNO$ has a perimeter of 45 centimeters. Triangle $MNO$ is dilated by a factor of $\frac{2}{5}$ to produce triangle $PQR$. What is the perimeter of triangle $PQR$?

A 18 cm  
B 9 cm  
C 10 cm  
D 15 cm

**Answer Key: page 239**

**Question 82**
A cylindrical tank has a volume of 300 gallons. A similar tank next to it has dimensions that are 3 times as large. What is the volume of the larger tank?

A 5400 gal  
B 2700 gal  
C 900 gal  
D 8100 gal

**Answer Key: page 239**

**Question 83**
A box shaped like a rectangular prism has a volume of 162 cubic inches. A smaller box has dimensions that are $\frac{2}{3}$ the dimensions of the larger box. What is the volume of the smaller box?

A 108 in.$^3$  
B 48 in.$^3$  
C 72 in.$^3$  
D 27 in.$^3$

**Answer Key: page 239**
Objective 9

The student will demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.

For this objective you should be able to

- identify proportional relationships in problem situations and solve problems;
- apply concepts of theoretical and experimental probability to make predictions;
- use statistical procedures to describe data; and
- evaluate predictions and conclusions based on statistical data.

How Do You Solve Problems Involving Proportional Relationships?

A ratio is a comparison of two quantities. A proportion is a statement that two ratios are equal. There are many real-life problems that involve proportional relationships. For example, you use proportions when converting units of measurement. You also use proportions to solve problems involving percents and rates.

To solve problems that involve proportional relationships, follow these guidelines:

- Identify the ratios to be compared. Be certain to compare the corresponding quantities, in the same order.
- Write a proportion, an equation in which the two ratios are set equal to each other.
- Solve the proportion. Use the fact that the cross products in a proportion are equal.

On a game show Mr. Williams answered 12 out of 15 questions correctly. What percent of the questions did Mr. Williams answer incorrectly?

- Mr. Williams answered 12 questions correctly. Therefore, he answered $15 - 12 = 3$ questions incorrectly.
- Write a ratio that compares the number of questions answered incorrectly to the total number of questions: $\frac{3}{15}$.
- To find the percent of questions answered incorrectly, find the part of 100, or the ratio $\frac{n}{100}$, that is equivalent to $\frac{3}{15}$.
- Write a proportion. $\frac{n}{100} = \frac{3}{15}$.
To solve for \( n \), use cross products and then divide by 15.

\[
15n = 3 \cdot 100 \\
15n = 300 \\
\frac{n}{15} = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}
\]

Mr. Williams answered 20% of the questions incorrectly.

---

**Try It**

At this time last year, Alfredo had $145 in his savings account. Today he has $152.98. If his savings continue to grow at the same rate, how much money will he have in his account at this time next year?

To find the rate at which Alfredo’s savings increased, divide the number of dollars by which his savings increased from last year to this year by $ \[ \boxed{145} \].

Alfredo’s savings increased by

\[
$152.98 - \boxed{145.00} = \boxed{7.98}.
\]

The rate by which his savings account grew is

\[
\frac{\boxed{7.98}}{\boxed{145}} = \boxed{0.055}.
\]

His savings should increase by \[ \boxed{5.5\%} \] over the next year.

Use a proportion to find 5.5% of his current balance, $152.98.

\[
\frac{5.5}{100} = \frac{x}{152.98} \\
\boxed{5.5} \times \boxed{x} = 5.5(\boxed{152.98}) \\
\boxed{5.5} \times \boxed{x} = \boxed{856.99} \\
\boxed{x} = \boxed{856.99}
\]

Alfredo’s savings account at this time next year should have $152.98 + \[ \boxed{856.99} = \boxed{1009.97} \].
His savings should increase by 5.5% over the next year.

\[ \frac{5.5}{100} = \frac{x}{152.98} \]
\[ 100x = 5.5(152.98) \]
\[ 100x = 841.39 \]
\[ x = 8.41 \]

Alfredo’s savings account at this time next year should have $152.98 + $8.41 = $161.39.

**What Is Probability?**

**Probability** is a measure of how likely an event is to occur. The probability of an event occurring is the ratio of the number of favorable outcomes to the number of all possible outcomes. In a probability experiment, favorable outcomes are the outcomes that you are interested in.

The probability, \( P \), of an event occurring must be from 0 to 1.

- If an event is impossible, its probability is 0.
- If an event is certain to occur, its probability is 1.

For example, the probability of drawing a blue marble from a bag containing 5 red marbles and 2 blue marbles is the ratio of the number of favorable outcomes, 2 blue marbles, to the number of possible outcomes, 7 marbles. The probability of drawing a blue marble is \( \frac{2}{7} \).

This is often written as \( P(\text{blue}) = \frac{2}{7} \).

A fair number cube has faces numbered 1 to 6. What is the probability of rolling an even number?

The sample space for this experiment is \{1, 2, 3, 4, 5, 6\}. There are a total of 6 possible outcomes.

A favorable outcome for this experiment is rolling a 2, 4, or 6. There are 3 favorable outcomes for this experiment.

The probability of rolling an even number is the ratio of the number of favorable outcomes to the number of possible outcomes:

\[ \frac{3}{6} = \frac{1}{2} \]
How Do You Find the Probability of Compound Events?

An event made up of a sequence of simple events is called a compound event. For example, flipping a coin and then rolling a number cube is a compound event.

One way to find the probability of a compound event is to multiply the probabilities of the simple events that make up the compound event.

If \( P(A) \) represents the probability of event \( A \) and \( P(B) \) represents the probability of event \( B \), then the probability of the compound event \( (A \text{ and } B) \) can be represented algebraically.

\[
P(A \text{ and } B) = P(A) \cdot P(B)
\]

For example, the probability of a fair coin landing heads up on one toss is \( P(H) = \frac{1}{2} \). The probability of a fair coin landing heads up on two tosses can be found as follows:

\[
P(H \text{ and } H) = P(H) \cdot P(H) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}
\]

When finding the probability of a compound event, you should first determine whether the simple events included are dependent or independent events.

- If the outcome of the first event affects the possible outcomes for the second event, the events are called dependent events.

  Suppose you draw 2 marbles, one at a time, from a bag with 4 white marbles and 1 red marble in it. You draw the second marble without replacing the first one you drew. Does the outcome of the first draw affect the likelihood of drawing a red marble on the second draw? Yes, because there were 5 marbles in the bag on the first draw, but only 4 on the second draw. The events are dependent.

- If the outcome of the first event does not affect the possible outcomes for the second event, the events are called independent events.

  Suppose you spin a spinner with the numbers 1 through 4 written on it, and then you spin it again. Does the outcome of the first spin affect the likelihood of spinning a 3 on the second spin? No. The events are independent.

A probability experiment consists of rolling a fair number cube with faces numbered 1 through 6 and then tossing a fair coin. What is the probability of rolling a 3 on the number cube and tossing tails on the coin?

- There are 6 possible outcomes for rolling the number cube: 1, 2, 3, 4, 5, and 6. There is only one favorable outcome, rolling a 3. The probability of rolling a 3 on the number cube is \( \frac{1}{6} \).

\[
P(3) = \frac{1}{6}
\]
- There are two possible outcomes for tossing the coin: heads (H) and tails (T). There is only one favorable outcome, T. The probability of tossing tails on the coin is \( \frac{1}{2} \).

\[ P(T) = \frac{1}{2} \]

- Find the probability of rolling a 3 on the number cube and tossing tails on the coin.

\[ P(3 \text{ and } T) = P(3) \cdot P(T) \]

\[ = \frac{1}{6} \cdot \frac{1}{2} \]

\[ = \frac{1}{12} \]

Another way to find the probability of this compound event is to look at the sample space for this experiment and identify the favorable outcomes. This method works only if the outcomes are all equally likely. Use a table to list all the possible outcomes. The one favorable outcome is shaded.

**Sample Space**

<table>
<thead>
<tr>
<th>Number Cube</th>
<th>Coin</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Heads</td>
</tr>
<tr>
<td>1</td>
<td>Tails</td>
</tr>
<tr>
<td>2</td>
<td>Heads</td>
</tr>
<tr>
<td>2</td>
<td>Tails</td>
</tr>
<tr>
<td>3</td>
<td>Heads</td>
</tr>
<tr>
<td>3</td>
<td>Tails</td>
</tr>
<tr>
<td>4</td>
<td>Heads</td>
</tr>
<tr>
<td>4</td>
<td>Tails</td>
</tr>
<tr>
<td>5</td>
<td>Heads</td>
</tr>
<tr>
<td>5</td>
<td>Tails</td>
</tr>
<tr>
<td>6</td>
<td>Heads</td>
</tr>
<tr>
<td>6</td>
<td>Tails</td>
</tr>
</tbody>
</table>

There are 12 possible outcomes, but only 1 of them is favorable. The probability of rolling a 3 on the number cube and tossing tails on the coin is \( \frac{1}{12} \).

This matches the result obtained using the rule for the probability of a compound event.

\[ P(3 \text{ and } T) = P(3) \cdot P(T) = \frac{1}{12} \]
A jar contains 4 red balls, 5 green balls, and 3 black balls. A ball is drawn, and then a second ball is drawn without replacing the first one. What is the probability of drawing two red balls in a row?

There are 12 balls in the jar on the first draw, and 11 balls in the jar on the second draw. The two events are dependent. The outcome of the first ball drawn affects the possible outcome of the second ball drawn.

- Find the probability of drawing a red ball on the first draw. There are 12 possible outcomes for the first draw. There are 4 favorable outcomes, 4 red balls.
  \[ P(\text{red first}) = \frac{4}{12} = \frac{1}{3} \]

- Find the probability of drawing a red ball on the second draw. There are now only 11 balls in the jar, so there are 11 possible outcomes for the second draw. Assume the first ball drawn was red. There are 3 favorable outcomes, 3 red balls left in the jar.
  \[ P(\text{red second}) = \frac{3}{11} \]

- Find the compound probability.
  \[ P(\text{red first and red second}) = P(\text{red first}) \cdot P(\text{red second}) = \frac{1}{3} \cdot \frac{3}{11} = \frac{3}{33} = \frac{1}{11} \]

The probability of drawing two red balls in a row is \( \frac{1}{11} \).

**What Is the Difference Between Theoretical and Experimental Probability?**

The theoretical probability of an event occurring is the ratio comparing the number of ways the favorable outcomes should occur to the number of all possible outcomes. If you toss a coin, theoretically the coin should land on heads \( \frac{1}{2} \) of the time.

\[ P(H) = \frac{1}{2} = 0.5 \]

The experimental probability of an event occurring is the ratio comparing the actual number of times the favorable outcome occurs in a series of repeated trials to the total number of trials. If you toss a coin 100 times, it is possible that the coin will land heads up 48 times and tails up 52 times. The experimental probability of the coin landing heads up in this situation would be \( \frac{48}{100} \).

\[ P(H) = \frac{48}{100} = 0.48 \]

The two types of probabilities, theoretical and experimental, are not always equal. In this case the theoretical probability is 0.5, but the experimental probability is 0.48.

For a given situation the experimental probability is usually close to, but slightly different from, the theoretical probability. The greater the number of trials, the closer the experimental probability should be to the theoretical probability.
A spinner in a game has 4 equal-sized regions: red, green, blue, and yellow. A group of players makes 40 trial spins and creates the table below.

<table>
<thead>
<tr>
<th>Color</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>9</td>
</tr>
<tr>
<td>Green</td>
<td>12</td>
</tr>
<tr>
<td>Blue</td>
<td>10</td>
</tr>
<tr>
<td>Yellow</td>
<td>9</td>
</tr>
</tbody>
</table>

How does the theoretical probability of landing on green compare to the experimental results?

- Since the colored regions are of equal size, the spinner is equally likely to land on any one of the colored regions. There are four possible outcomes for this experiment: red, green, blue, and yellow. There is one favorable outcome: green. The theoretical probability of landing on the green is \( \frac{1}{4} \), or \( P(\text{green}) = 0.25 \).
- The experimental probability of landing on green is the ratio of the number of times the spinner landed on green in the experiment to the total number of spins. The spinner landed on green 12 times out of 40 spins. The experimental probability of landing on green is \( \frac{12}{40} = \frac{3}{10} \), or \( P(\text{green}) = 0.3 \).

The experimental probability, 0.3, and the theoretical probability, 0.25, are close to each other but not equal.

**How Do You Use Probability to Make Predictions and Decisions?**

You can use either theoretical or experimental probabilities to make predictions. If you know the probability of an event occurring and you also know the total number of trials, then you can predict the likely number of favorable outcomes.

- Write a ratio that represents the probability of an event occurring.
- Write a ratio that compares the number of favorable outcomes to the number of trials.
- Write a proportion.
- Solve the proportion.
A manufacturer tests 125 lightbulbs to determine the number of hours the lightbulbs last. The results are shown in the table below.

<table>
<thead>
<tr>
<th>Outcome (hours)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>950–974</td>
<td>15</td>
</tr>
<tr>
<td>975–999</td>
<td>20</td>
</tr>
<tr>
<td>1,000–1,024</td>
<td>50</td>
</tr>
<tr>
<td>1,025–1,049</td>
<td>40</td>
</tr>
</tbody>
</table>

Based on the experimental results, how many lightbulbs from a shipment of 50,000 lightbulbs should be expected to last at least 1,000 hours?

- Find the experimental probability that a lightbulb will last at least 1,000 hours.

  Based on the table, a total of $50 + 40 = 90$ lightbulbs lasted at least 1,000 hours. There were a total of 125 trials. The experimental probability that a lightbulb will last at least 1,000 hours is $\frac{90}{125} = \frac{18}{25}$.

- Write a proportion.

  $\frac{18}{25} = \frac{x}{50,000}$

- Solve by using cross products and dividing by 25.

  $25x = 18 \cdot 50,000$
  $25x = 900,000$
  $x = 36,000$

Therefore, 36,000 lightbulbs from the shipment of 50,000 lightbulbs should be expected to last at least 1,000 hours.

---

**Try It**

Some sophomores were surveyed about which candidate they plan to vote for in the upcoming class-representative election. The results of the survey are shown below.

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alex</td>
<td>16</td>
</tr>
<tr>
<td>Terry</td>
<td>8</td>
</tr>
<tr>
<td>Alicia</td>
<td>11</td>
</tr>
</tbody>
</table>

If there are 315 sophomores in the school, what is the best estimate of the number of votes Alicia will receive?
Objective 9

Find the probability of a sophomore voting for Alicia. Of the ______ students who were surveyed, ______ said they plan to vote for Alicia. The experimental probability of a sophomore voting for Alicia is ______. There are ______ sophomores in the school. Let $n$ represent the number of sophomores who will vote for ______.

Write a proportion and solve.

$$\frac{n}{35} = \frac{11}{35} \cdot \frac{35n}{35} = \frac{3465}{35}$$

$$n = 99$$

Based on the survey, the best estimate of the number of votes Alicia will receive is 99.

---

Find the experimental probability of a sophomore voting for Alicia. Of the 35 students who were surveyed, 11 said they plan to vote for Alicia. The experimental probability of a sophomore voting for Alicia is $\frac{11}{35}$. There are 315 sophomores in the school. Let $n$ represent the number of sophomores who will vote for Alicia.

$$\frac{n}{35} = \frac{11}{35}$$

$$35n = 11 \cdot 315$$

$$35n = 3465$$

$$n = \frac{3465}{35}$$

$$n = 99$$

Based on the survey, the best estimate of the number of votes Alicia will receive is 99.
### How Do You Use Mode, Median, Mean, and Range to Describe Data?

There are many ways to describe the characteristics of a set of data. The mode, median, and mean are all called **measures of central tendency**.

<table>
<thead>
<tr>
<th><strong>Mode</strong></th>
<th>The <strong>mode</strong> of a set of data describes which value occurs most frequently. If two or more numbers occur the same number of times and more often than all the other numbers in the set, those numbers are all modes for the data set. If each of the numbers in a set occurs the same number of times, the set of data has no mode.</th>
<th>Use the mode to show which value or values in a set of data occur most often. For the set ( {1, 4, 9, 3, 1, 6} ) the mode is 1 because it occurs most frequently. The set ( {1, 4, 3, 3, 1, 6} ) has two modes, 1 and 3, because they both occur twice and most frequently.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Median</strong></td>
<td>The <strong>median</strong> of a set of data describes the middle value when the set is ordered from greatest to least or from least to greatest. If there are an even number of values, the median is the average of the two middle values. Half the values are greater than the median, and half the values are less than the median. The median is a good measure of central tendency to use when a set of data has an outlier, a number that is very different in value from the other numbers in the set.</td>
<td>Use the median to show which number in a set of data is in the middle when the numbers are listed in order. For the set ( {1, 4, 9, 3, 1, 6} ) the median is 4 because it is in the middle when the numbers are listed in order: ( {1, 3, 4, 6, 9} ). For the set ( {1, 4, 9, 3, 1, 6} ) the median is ( \frac{3 + 4}{2} = 3.5 ) because 3 and 4 are in the middle when the numbers are listed in order: ( {1, 1, 3, 4, 6, 9} ). Their values must be averaged to find the median.</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>The <strong>mean</strong> of a set of data describes the average of the numbers. To find the mean, add all the numbers and then divide by the number of items in the set. The mean of a set of data can be greatly affected if one of the numbers is an outlier, a number that is very different in value from the other numbers in the set. The mean is a good measure of central tendency to use when a set of data does not have any outliers.</td>
<td>Use the mean to show the numerical average of a set of data. For the set ( {1, 4, 9, 3, 1, 6} ) the mean is the sum, 24, divided by the number of items, 6. The mean is ( 24 \div 6 = 4 ).</td>
</tr>
<tr>
<td><strong>Range</strong></td>
<td>The <strong>range</strong> of a set of data describes how big a spread there is from the largest value in the set to the smallest value.</td>
<td>Use the range to show how much the numbers vary. For the set ( {1, 4, 9, 3, 1, 6} ) the range is ( 9 - 1 = 8 ).</td>
</tr>
</tbody>
</table>

To decide which of these measures to use to describe a set of data, look at the numbers and ask yourself, *What am I trying to show about the data?*
Each night for one week, a restaurant manager recorded the number of customers who came for dinner. The results are shown in the table below.

<table>
<thead>
<tr>
<th>Night</th>
<th>Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunday</td>
<td>58</td>
</tr>
<tr>
<td>Monday</td>
<td>57</td>
</tr>
<tr>
<td>Tuesday</td>
<td>60</td>
</tr>
<tr>
<td>Wednesday</td>
<td>55</td>
</tr>
<tr>
<td>Thursday</td>
<td>65</td>
</tr>
<tr>
<td>Friday</td>
<td>66</td>
</tr>
<tr>
<td>Saturday</td>
<td>149</td>
</tr>
</tbody>
</table>

Which measure of the data would best describe the number of customers who eat in the restaurant on a typical night?

- The range is the difference between the largest value and the smallest: $149 - 55 = 94$. The range does not describe the number of customers who eat in the restaurant on a typical night.

- Each of the numbers in the set of data occurs only once. The set of data has no mode.

- The mean number of customers is approximately 73. However, the number of customers on Saturday night, 149, is much higher than the number of customers on any other night. This data point is an outlier.

In this case, the mean does not give a very good representation of the number of customers who eat in the restaurant on a typical night; it is too high.

- The median number of customers is found by listing the number of customers in order and finding the middle value. Listed in order, the numbers are $\{55, 57, 58, 60, 65, 66, 149\}$. The middle number is 60. The median, 60, is not affected by the outlier.

In this case, the median best describes the number of customers who eat in the restaurant on a typical night.
A science club sold candy bars to raise money. The table shows the number of candy bars sold during the first five days of the sale.

<table>
<thead>
<tr>
<th>Day</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>103</td>
</tr>
<tr>
<td>2</td>
<td>115</td>
</tr>
<tr>
<td>3</td>
<td>122</td>
</tr>
<tr>
<td>4</td>
<td>117</td>
</tr>
<tr>
<td>5</td>
<td>117</td>
</tr>
</tbody>
</table>

On the sixth day of the sale, the club sold only 56 candy bars. How does the significantly smaller number of candy bars sold on the sixth day affect the different measures of the data?

**Range**
The range is the difference between the largest value and the smallest.
- For the first 5 days, the range is $122 - 103 = 19$.
- With the sixth day included, the range is $122 - 56 = 66$.

The range of values increases significantly because the sixth value, an outlier, is well outside the previous range.

**Median**
The median is found by listing the number of candy bars sold in order and picking the middle value.
- For the first 5 days: $103, 115, 117, 117, 122$
  
  $\text{median} = 117$
- With the sixth day included: $56, 103, 115, 117, 117, 122$
  
  $\text{median} = (115 + 117) ÷ 2 = 116$

The median value is not significantly affected by the outlier.

**Mode**
The mode of a set of numbers tells which value occurs most frequently.
- For the first 5 days, the mode is 117.
- With the sixth day included, the mode is still 117.

The mode is not affected by the outlier.

**Mean**
The mean is the average of the values.
- For the first 5 days:
  
  $$\frac{103 + 115 + 122 + 117 + 117}{5} = 114.8$$

- With the sixth day added, the mean is:
  
  $$\frac{103 + 115 + 122 + 117 + 117 + 56}{6} = 105$$

Since 56 is much smaller than the other values, it significantly lowers the mean.
How Do You Use Graphs to Represent Data?

There are many ways to represent data graphically. Bar graphs, histograms, and circle graphs are three types of graphs used to display data. Graphical representations of data often make it easier to see relationships in the data. However, if the conclusions drawn from a graph are to be valid, you must read and interpret the data from the graph accurately.

A bar graph uses bars of different heights or lengths to show the relationships between different groups or categories of data.

The bar graph below shows the number of people in a town who attended one of two different movies, A or B, on each of six days last week. What conclusions can you draw about the number of people attending each of the movies each day?

- The graph shows that more people attended each of the two movies on Saturday than on any other day.
- For the week, more people attended Movie B than Movie A.
- The least number of people attended Movie B than Movie A.
- The least number of people attended either movie on Wednesday.
- The combined attendance at both movies on Monday and Thursday was the same.
A histogram is a special kind of bar graph that shows the number of data points that fall within specific intervals of values. The intervals into which the data’s range is divided should be equal. If the intervals are not equal, the graph could be misleading and result in invalid conclusions.

Look at the histogram below showing the number of dogs of various weights in a kennel.

The histogram shows the ages of people attending a symphony performance. Their ages are divided into equal 10-year intervals.

What conclusions can you draw from the graph about attendance at the performance?

The broken line on the vertical axis means that attendance values from 0 through 9 are not shown in the graph. Using the broken line allows the graph to have shorter bars. Because of this, the lengths of the bars should not be used to make direct comparisons. Instead, read the values from the graph and compare them.

- A total of 24 people between the ages of 30 and 39 attended the performance. This age group had the greatest number of people in attendance.
Objective 9

A total of 20 people between the ages of 20 and 29 attended the performance, and 10 people between the ages of 0 and 9 attended. Twice as many people between the ages of 20 and 29 attended the performance as people between the ages of 0 and 9.

If you had compared the bar lengths for these two data intervals you could have reached an invalid conclusion. The bar for one is about five times as large as for the other, yet the numerical value is only twice as large.

A circle graph represents a set of data by showing the relative size of the parts that make up the whole. The circle represents the whole, or the sum of all the data elements. Each section of the circle represents a part of the whole. The number of degrees in the central angle of the section should be proportional to the number of degrees in a circle, 360°.

Suppose you were constructing a circle graph that compared the number of school-sponsored sports teams on which juniors in your school play. Their participation data are shown in the table below.

<table>
<thead>
<tr>
<th>Juniors Playing Sports</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Sports</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3 or more</td>
</tr>
</tbody>
</table>

What size central angle should be used for the sector of the circle used to represent the students who play no sports?

One way to solve this problem is to first find the percent of the circle that should be used to represent the students who play no sports.

- The table represents a total of 200 students. Since 80 of them play no sports, the fraction of students who play no sports is \( \frac{80}{200} \).
- Convert \( \frac{80}{200} \) to a percent.
  \[
  \frac{80}{200} = \frac{40}{100} = 40\%
  \]
- So, 40% of the students play no sports. Therefore, 40% of the circle should be used to represent students who play no sports.

Next, find what central angle should be used for the sector of the circle used to represent students who play no sports. Since a circle has 360°, find 40% of 360°.

\[
\frac{40}{100} = \frac{n}{360}
\]

\[
100n = 14,400
\]

\[
n = 144
\]
A central angle of $144^\circ$ should be used for the sector of the circle used to represent the students who play no sports.

Customers at a grocery store were asked which brand of soft drink they prefer. The results of the survey are shown in the circle graph below.

What conclusions can you draw about the soft-drink preferences of the customers at the store?

- The graph shows that the most preferred brand is Brand A, 38%.
- The graph shows that 25%, or $\frac{1}{4}$, of the customers surveyed preferred Brand D.
- The smallest fraction of customers, 10%, had no preference in soft drinks.
The graph below shows the average populations of three cities during three five-year intervals.

For which city was the increase in population the greatest between 1985 and 1999?

- The population of City C decreased between 1985 and 1999.
- The population of City B increased between 1985 and 1999. Find the increase in population. The lowest average population, approximately 75,000, was during the period 1985–1989, and the highest average population, approximately 110,000, was during the period 1995–1999. The approximate increase in population was 110,000 – 75,000 = 35,000 people.
- The population of City A increased between 1985 and 1999. Find the increase in population. The lowest average population, approximately 30,000, was during the period 1985–1989, and the highest average population, approximately 80,000, was during the period 1995–1999. The approximate increase in population was 80,000 – 30,000 = 50,000 people.

The increase in population between 1985 and 1999 was the greatest for City A.

Now practice what you’ve learned.
Question 84
Rhonda estimated it would take 12 hours to complete her research project. If this represents only 80% of the number of hours it actually took her to complete the project, how many hours did Rhonda spend on the project?

A 9.6 h  
B 15 h  
C 3 h  
D 960 h

Question 85
A factory worker can manufacture 35 electronic switches in 1.5 hours. At this rate, how many hours will it take him to manufacture 210 switches?

A 6 h  
B 9 h  
C 4900 h  
D 140 h

Question 86
Jonathan draws two tickets from a box to select the door-prize winners at a party. The tickets are numbered from 1 to 25. What is the probability that both of the tickets drawn will have numbers less than 5?

A $\frac{1}{50}$  
B $\frac{2}{75}$  
C $\frac{12}{625}$  
D $\frac{1}{5}$

Question 87
Reggie is a professional baseball player. He has the following batting record.

<table>
<thead>
<tr>
<th>Type of Hit</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Singles</td>
<td>210</td>
</tr>
<tr>
<td>Doubles</td>
<td>20</td>
</tr>
<tr>
<td>Triples</td>
<td>1</td>
</tr>
<tr>
<td>Home runs</td>
<td>6</td>
</tr>
<tr>
<td>No hits</td>
<td>574</td>
</tr>
</tbody>
</table>

Based on this record, what is the probability that Reggie will get a hit during his next time at bat?

A 0.413  
B 0.186  
C 0.292  
D 0.366
**Objective 9**

**MATHEMATICS**

**Question 88**

A horticulturist selected a sample of seeds from a crop. She planted the seeds, and after 3 months she measured the height of each plant to the nearest eighth of an inch. The results are shown in the table below.

<table>
<thead>
<tr>
<th>Height (inches)</th>
<th>Number of Plants</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–1(\frac{7}{8})</td>
<td>9</td>
</tr>
<tr>
<td>2–3(\frac{7}{8})</td>
<td>16</td>
</tr>
<tr>
<td>4–5(\frac{7}{8})</td>
<td>26</td>
</tr>
<tr>
<td>6 or more</td>
<td>24</td>
</tr>
</tbody>
</table>

Based on the results in the table, how many seeds out of 600 could the horticulturist expect to reach a height of at least 2 inches in 3 months?

A  450  
B  550  
C  528  
D  384  

---

**Question 89**

Jean is a member of the school’s bowling club. At the last practice session, each team member bowled 3 games and recorded his or her score on a master list. Which measure of the data should Jean use if she wants to identify the score that has as many scores below it as above it?

A  Mean  
B  Mode  
C  Median  
D  Range  

**Answer Key:** page 240
Question 90

The table below represents Trish’s expenses for a scuba-diving vacation in the Caribbean.

<table>
<thead>
<tr>
<th>Expense</th>
<th>Amount (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane fare</td>
<td>600</td>
</tr>
<tr>
<td>Food</td>
<td>200</td>
</tr>
<tr>
<td>Hotel</td>
<td>360</td>
</tr>
<tr>
<td>Cab fares</td>
<td>40</td>
</tr>
<tr>
<td>Gifts</td>
<td>50</td>
</tr>
<tr>
<td>Dive boat</td>
<td>250</td>
</tr>
</tbody>
</table>

Trish’s Scuba Vacation

Which graph matches the information in the table?

A

B

C

D
Objective 9

MATHEMATICS

Question 91

The graph below represents car sales at a dealership for the first four months of the year.

Which of the tables below represents the data in the graph?

<table>
<thead>
<tr>
<th>Month</th>
<th>Cars Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>900</td>
</tr>
<tr>
<td>February</td>
<td>2400</td>
</tr>
<tr>
<td>March</td>
<td>2900</td>
</tr>
<tr>
<td>April</td>
<td>2400</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Month</th>
<th>Cars Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1500</td>
</tr>
<tr>
<td>February</td>
<td>2900</td>
</tr>
<tr>
<td>March</td>
<td>3100</td>
</tr>
<tr>
<td>April</td>
<td>2400</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Month</th>
<th>Cars Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1100</td>
</tr>
<tr>
<td>February</td>
<td>2400</td>
</tr>
<tr>
<td>March</td>
<td>3100</td>
</tr>
<tr>
<td>April</td>
<td>2600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Month</th>
<th>Cars Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1100</td>
</tr>
<tr>
<td>February</td>
<td>1900</td>
</tr>
<tr>
<td>March</td>
<td>2900</td>
</tr>
<tr>
<td>April</td>
<td>2400</td>
</tr>
</tbody>
</table>
**Question 92**

The total number of hours Ava spent studying each week for the first 8 weeks of school are shown in the table below.

<table>
<thead>
<tr>
<th>Week</th>
<th>Hours Studying</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

Which measure should Ava use to show the number of hours she most frequently spends studying in a school week?

A  Mean  
B  Median  
C  Mode  
D  Range

**Question 93**

The graph shows CD sales at a music store for 6 consecutive weeks.

Based on the data in the graph, which of the following conclusions is true?

A  Sales increased at a constant rate each week over the six-week period.  
B  The store sold an average of approximately 289 CDs per week over the six-week period.  
C  Three times as many CDs were sold during the sixth week as during the first week.  
D  The store sold more than 1800 CDs during the six-week period.
Objective 10

The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

For this objective you should be able to
- apply mathematics to everyday experiences and activities;
- communicate about mathematics; and
- use logical reasoning.

How Do You Apply Math to Everyday Experiences?

Suppose you want to compute the likelihood of winning an election based on the results of a random survey. Or suppose you need to estimate the volume of a container based on its dimensions. Finding the solution to problems such as these often requires the use of math.

Solving problems involves more than just arithmetic; logical reasoning and careful planning also play very important roles. The steps in problem solving include understanding the problem, making a plan, carrying out the plan, and evaluating the solution to determine whether it is reasonable.

The Bradley family is planning a summer vacation. They expect to drive a total of 250 to 300 miles. Their car gets 20 miles per gallon of gasoline, and gas is expected to cost from $1.35 per gallon to $1.55 per gallon. Based on this information, what is the least amount the Bradley family should expect to spend on fuel for their vacation? What is the greatest amount?

- What information is given?
  - length of trip: 250 to 300 miles
  - gas mileage: 20 mpg
  - cost of gas: $1.35/gal to $1.55/gal

- What do you need to find?
  You need to find the number of gallons of gas the Bradleys could use and, based on those figures, the total cost of the gasoline.
Objectives

1. Find the number of gallons of gas that could be used—minimum and maximum amounts.
   Divide the number of miles to be traveled by the rate the gas will be used, 20 mpg.
   Calculate first using the minimum figure for the length of the trip, 250 miles, and then using the maximum, 300 miles.

2. Find the minimum and maximum possible cost of the gasoline.
   Multiply the cost of gasoline by the number of gallons to be used.
   Calculate first using the minimum cost of the gasoline, $1.35/gal, and then using the maximum, $1.55/gal.

3. Find the number of gallons of gas that will be used.

<table>
<thead>
<tr>
<th>Minimum Calculation</th>
<th>Maximum Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 miles</td>
<td>300 miles</td>
</tr>
<tr>
<td>250 miles ÷ 20 mpg</td>
<td>300 miles ÷ 20 mpg</td>
</tr>
<tr>
<td>= 12.5 gallons</td>
<td>= 15 gallons</td>
</tr>
</tbody>
</table>

4. Find the cost of the gas that will be used.

<table>
<thead>
<tr>
<th>Minimum Calculation</th>
<th>Maximum Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5 gallons at $1.35/gal</td>
<td>15 gallons at $1.55/gal</td>
</tr>
<tr>
<td>12.5 gallons • $1.35/gal</td>
<td>15 gallons • $1.55/gal</td>
</tr>
<tr>
<td>= $16.88</td>
<td>= $23.25</td>
</tr>
</tbody>
</table>

The least amount the Bradley family should expect to spend on gasoline for their vacation is $16.88, and the greatest amount is $23.25.
A group of hikers at a state park follow the trail shown on the map below. They hike from the parking lot to the waterfall and then to a scenic overlook. From the overlook the group hikes back to the parking lot.

If 1 inch on the map equals 2 miles, to the nearest tenth of a mile how far did the group hike?

What information are you given in the problem?

The distance from the waterfall to the overlook is ______ inches on the map.

The distance from the overlook to the parking lot is ______ inch on the map.

The scale used on the map is ______ inch = ______ miles.

You need to find the distance in miles from the parking lot to the waterfall.

The hikers' path forms a ________________________ triangle.

The distance from the parking lot to the waterfall is the __________________________ of the triangle.

Find this distance on the map using the Pythagorean Theorem.

__________ this distance to the two known distances on the map.

Use the map's scale to find the actual distance the group hiked.
Substitute the values for a and b into the Pythagorean Theorem and solve for c.

\[ a^2 + b^2 = c^2 \]
\[ 1^2 + 2^2 = c^2 \]
\[ 1 + 4 = c^2 \]
\[ 5 = c^2 \]
\[ \sqrt{5} = c \]
\[ c \approx 2.24 \text{ inches} \]

On the map the total distance of the hike is

\[ a + b + c \approx \text{____ inches} \]

Use the scale to convert the distance to miles.

\[ \frac{2.24 \text{ inches}}{5.24 \text{ inches}} \approx \text{____ miles} \]

The question asks for the total distance to the nearest tenth of a mile.

The total distance the group hiked is ____ miles.

The distance from the waterfall to the overlook is 2 inches on the map.
The distance from the overlook to the parking lot is 1 inch on the map.
The scale used on the map is 1 inch = 2 miles.
The hikers’ path forms a right triangle. The distance from the parking lot to the waterfall is the hypotenuse of the triangle. Add this distance to the two known distances on the map.

\[ a^2 + b^2 = c^2 \]
\[ 1^2 + 2^2 = c^2 \]
\[ 5 = c^2 \]
\[ \sqrt{5} = c \]
\[ c \approx 2.24 \text{ inches} \]

On the map the total distance of the hike is \[ 2.24 + 1 \approx 3.24 \text{ inches} \].
The distance in miles is \[ 5.24 \cdot 2 \approx 10.48 \approx 10.5 \text{ miles} \]. The question asks for the total distance to the nearest tenth of a mile. The total distance the group hiked is 10.5 miles.
What Is a Problem-Solving Strategy?

A problem-solving strategy is a plan for solving a problem. Different strategies work better for different types of problems. Sometimes you can use more than one strategy to solve a problem. As you practice solving problems, you will discover which strategies you prefer and which work best in various situations.

Some problem-solving strategies include

- drawing a picture;
- looking for a pattern;
- guessing and checking;
- acting it out;
- making a table;
- working a simpler problem; and
- working backwards.

Alfonso is pouring liquid gelatin from a container into four small gelatin molds. The liquid gelatin is in a cylindrical container with a diameter of 10 centimeters. It fills the container to a level of 15.9 cm. Each of the gelatin molds is a rectangular prism that is 5 cm by 7.5 cm by 8.1 cm. Will Alfonso be able to transfer all of the gelatin from the cylindrical container into the four molds? If not, how many cubic centimeters of gelatin will remain after he fills the molds?

Draw a picture of the containers and label the dimensions.

- The dimensions of the cylinder and the dimensions of the prisms are given in the problem.
- You need to find the volume of the cylinder and the combined volume of the four rectangular prisms.
Find the volume of the cylinder. Use the formula for the area of a circle, \( A = \pi r^2 \), to find the area of the circular base. The diameter is 10 cm, so the radius, \( r \), is 5 cm. Use the area of the base to calculate the volume of the gelatin using the formula \( V = Bh \).

\[
\begin{align*}
A &= \pi r^2 \\
A &= \pi \cdot 5^2 \\
A &= \pi \cdot 25 \\
A &\approx 78.5 \text{ cm}^2
\end{align*}
\]

\[
\begin{align*}
V &= Bh \\
V &\approx 78.5 \cdot 15.9 \\
V &\approx 1248.15 \text{ cm}^3
\end{align*}
\]

The volume of the gelatin is approximately 1248 cm\(^3\).

Find the combined volume of the four rectangular prisms. The area of the rectangular base equals its length times its width.

\[
\begin{align*}
B &= 5 \cdot 7.5 = 37.5 \text{ cm}^2 \\
V &= Bh \\
V &= 37.5 \cdot 8.1 \\
V &= 303.75 \text{ cm}^3
\end{align*}
\]

The total volume of the four smaller containers is \( 4 \cdot 303.75 = 1215 \text{ cm}^3 \).

Since 1215 < 1248, the liquid gelatin will not fit in the four containers. There will be about 33 cubic centimeters (1248 − 1215) of liquid gelatin remaining.

At a business meeting each person in the room shakes hands with every other person. If there are 5 people at the meeting, how many handshakes take place?

One way to solve this problem is to draw a picture.

Draw 5 points representing the 5 people.

To model the handshakes, draw a line segment connecting each point to the four other points. Each point represents one person. Count the number of line segments. There are 10 line segments. At the meeting, ten handshakes take place.
A store sells boxes of candy wrapped in decorative paper. Each box is in the shape of a rectangular prism with the following dimensions: length $6 \frac{5}{8}$ inches, width $4 \frac{1}{4}$ inches, and height $2 \frac{15}{16}$ inches. Wrapping a box requires about 20% more paper than the box's surface area. Approximately how many square feet of paper would be needed to wrap 200 boxes of candy?

Estimate the answer by rounding the dimensions of the box to the nearest inch.

\[
6 \frac{5}{8} \text{ inches} \approx _____ \text{ inches}
\]
\[
4 \frac{1}{4} \text{ inches} \approx _____ \text{ inches}
\]
\[
2 \frac{15}{16} \text{ inches} \approx _____ \text{ inches}
\]

Find the surface area of one candy box.

Each surface is shaped like a ________________.

Find the surface area by finding the ________________ of the areas of all the surfaces.

\[
S = 2(____\cdot____) + 2(____\cdot____) + 2(____\cdot____) \\
= _____ \text{ square inches}
\]

Multiply by _____ to find the combined surface area of all the boxes.

____ in.$^2 \cdot _____ = _____ \text{ in.}^2$

There are 12 inches in 1 foot and 144 square inches in 1 square foot.

Divide by _____ to convert the surface area to square feet.

______ in.$^2 \div 144 = _____ \approx 170 \text{ ft}^2$

It takes 20% more paper to wrap a box than its surface area. Find 20% of _____.

\[
20\% \text{ of } _____ = 0.20(_____) = _____ \text{ ft}^2
\]

Find the total number of square feet of paper needed to wrap the candy boxes.

_____ ft$^2$ + _____ ft$^2$ = _____ ft$^2$

The store would need approximately _____ square feet of paper to wrap 200 candy boxes.
Each surface is shaped like a rectangle. Find the surface area by finding the sum of the areas of all the surfaces.

\[ S = 2(7 \cdot 4) + 2(4 \cdot 3) + 2(7 \cdot 3) = 122 \text{ square inches} \]

Multiply by 200 to find the combined surface area of all the boxes.

\[ 122 \text{ in.}^2 \cdot 200 = 24,400 \text{ in.}^2 \]

Divide by 144 to convert the surface area to square feet.

\[ 24,400 \text{ in.}^2 \div 144 = 169.44 = 170 \text{ ft}^2 \]

It takes 20% more paper to wrap a box than its surface area. Find 20% of 170.

\[ 20\% \text{ of } 170 = 0.20(170) = 34 \text{ ft}^2 \]

\[ 170 \text{ ft}^2 + 34 \text{ ft}^2 = 204 \text{ ft}^2 \]

The store would need approximately 204 square feet of paper to wrap 200 candy boxes.
How Do You Communicate About Mathematics?

It is important to be able to rewrite a problem using mathematical language and symbols. The words in the problem will give clues about the operations that you will need in order to solve the problem. In some problems it may be necessary to use algebraic symbols to represent quantities and then use equations to express the relationships between the quantities. In other problems you may need to represent the given information using a table or graph.

The function \( h = -16t^2 + 400 \) represents the height in feet, \( h \), of an object dropped from a height of 400 feet in terms of \( t \), the number of seconds it falls. The following graph represents this relationship.

Why is the graph of this relationship restricted to Quadrant I?

All points in Quadrants II, III, and IV have at least one negative coordinate. If this graph were extended into those quadrants, then either an \( h \) or \( t \) value would be negative, or both.

In this problem the variable \( t \) can only be positive because it represents the number of seconds during which the object falls. The number of seconds begins at 0. It ends at some positive value, the number of seconds when the object hits the ground. The number of seconds cannot be negative.

In this problem the variable \( h \) can only have positive values because it represents the height of the object above the ground. Height begins at 400 feet and ends at 0 feet, the height of the object when it hits the ground. The object is never below the ground, so \( h \) is never negative.

Since neither \( t \) nor \( h \) can be negative, this graph cannot be drawn in Quadrants II, III, or IV.
Try It

The following table summarizes the number of points scored per game by a player on the Central High School boys’ basketball team for the first 10 games of the season.

<table>
<thead>
<tr>
<th>Game</th>
<th>Points Scored</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>26</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>10</td>
<td>18</td>
</tr>
</tbody>
</table>

If the scoring record above is used to predict the number of points he is likely to score at his next game, what is the probability that he will score more points than his mean score for the first 10 games?

His mean score to date is the total number of points he has scored divided by the number of games he has played.

\[
\frac{12 + 18 + 26 + 15 + 8 + 12 + 20 + 14 + 13 + 18}{10} = \frac{156}{10} = 15.6
\]

To find the probability of scoring above his mean score, find the number of favorable outcomes, the number of games in which he scored above his mean.

In _____ out of the last 10 games, he scored over 15.6 points.

The probability that he will score more than 15.6 points is \( \frac{4}{10} \).
How Do You Use Logical Reasoning as a Problem-Solving Tool?

You can use logical reasoning to find patterns in a set of data. You can then use those patterns to draw conclusions about the data.

Finding patterns involves identifying characteristics that objects or numbers have in common. Look for the pattern in different ways. A sequence of geometric objects may have some property in common. For example, they may all be rectangular prisms or all be dilations of the same object.

The following table shows a series of dilations of a rectangle.

<table>
<thead>
<tr>
<th>Dilation</th>
<th>Length (inches)</th>
<th>Width (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>108</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>86.4</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>60.48</td>
<td>25.2</td>
</tr>
</tbody>
</table>

By what scale factor would the next rectangle in the series be dilated? To find the pattern, you must first find the scale factor used in each dilation. To do so, find the ratio of corresponding sides.

<table>
<thead>
<tr>
<th>Dilation</th>
<th>Length (inches)</th>
<th>Width (inches)</th>
<th>Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>108</td>
<td>45</td>
<td>( \frac{108}{120} = \frac{45}{50} = 0.9 )</td>
</tr>
<tr>
<td>3</td>
<td>86.4</td>
<td>36</td>
<td>( \frac{86.4}{108} = \frac{36}{45} = 0.8 )</td>
</tr>
<tr>
<td>4</td>
<td>60.48</td>
<td>25.2</td>
<td>( \frac{60.48}{86.4} = \frac{25.2}{36} = 0.7 )</td>
</tr>
</tbody>
</table>

The scale factors are decreasing by 0.1 at each step. If the pattern continues, the scale factor for the next dilation should be \( 0.7 - 0.1 = 0.6 \).
Try It

The graph below summarizes the percent of students from each grade who voted for Candidate A and Candidate B, the top two candidates in a school election.

If the voting pattern established by the 9th, 10th, and 11th graders continues, how many votes should Candidate B expect to get from the 320 twelfth graders who voted?

Look for a pattern in the data.

The percent of 9th-grade students voting for Candidate B was _____% more than the percent voting for Candidate A.

The percent of 10th-grade students voting for Candidate B was _____% more than the percent voting for Candidate A.

The percent of 11th-grade students voting for Candidate B was _____% more than the percent voting for Candidate A.

If the pattern continues, the percent of 12th-grade students voting for Candidate B should be _____% more than the percent voting for Candidate A.
If _____% of the 12th graders voted for Candidate A, then _____% + 20% = _____% should vote for Candidate B.

Find _____% of 320 students. Write a proportion.

\[
\frac{100}{100} = \frac{x}{320}
\]

100x = _____ • _____
100x = ______
\[x = ______\]

Candidate B should get _____ votes from the 12th graders.

The percent of 9th-grade students voting for Candidate B was 5% more than the percent voting for Candidate A. The percent of 10th-grade students voting for Candidate B was 10% more than the percent voting for Candidate A. The percent of 11th-grade students voting for Candidate B was 15% more than the percent voting for Candidate A.

If the pattern continues, the percent of 12th-grade students voting for Candidate B should be 20% more than the percent voting for Candidate A. If 25% of the 12th graders voted for Candidate A, then 25% + 20% = 45% should vote for Candidate B. Find 45% of 320 students.

\[
\frac{45}{100} = \frac{x}{320}
\]

100x = 45 • 320
100x = 14,440
\[x = 144\]

Candidate B should get 144 votes from the 12th graders.
The solution to a problem can be justified by identifying the mathematical properties or relationships that produced the answer. You should have a reason for drawing a conclusion, and you should be able to explain that reason.

The table below shows the number of people who owned homes in Williamstown for several different years.

<table>
<thead>
<tr>
<th>Home Ownership in Williamstown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>1998</td>
</tr>
<tr>
<td>1999</td>
</tr>
<tr>
<td>2000</td>
</tr>
<tr>
<td>2001</td>
</tr>
</tbody>
</table>

Did the number of people who owned homes increase by the same percent each year during this period?

First check whether the number of people who owned homes increased each year. Then check whether the percent increase is the same for each year.

- To find the percent increase from 1998 to 1999, subtract the number of people owning homes in 1998 from the number owning homes in 1999. Then divide by the number owning homes in 1998.
  
  \[
  \text{Percent increase} = \frac{24,750 - 22,500}{22,500} = \frac{2,250}{22,500} = 0.10 = 10\% 
  \]

- To find the percent increase from 1999 to 2000, subtract the number of people owning homes in 1999 from the number owning homes in 2000. Then divide by the number owning homes in 1999.
  
  \[
  \text{Percent increase} = \frac{27,225 - 24,750}{24,750} = \frac{2,475}{24,750} = 0.10 = 10\% 
  \]

- To find the percent increase from 2000 to 2001, subtract the number of people owning homes in 2000 from the number owning homes in 2001. Then divide by the number owning homes in 2000.
  
  \[
  \text{Percent increase} = \frac{28,300 - 27,225}{27,225} = \frac{1,075}{27,225} \approx 0.0395 \approx 4\% 
  \]

The percent increase from 2000 to 2001 is less than the percent increase for the previous years. The number of people who owned homes did not increase by the same percent each year.

Now practice what you’ve learned.
Objective 10

**Question 94**

Vickie is comparing the cost of a DVD player at two different stores. The DVD player costs $119 at Store A and $138 at Store B. Vickie has a 15%-off coupon for Store B, and Store A is having a 10%-off sale. Which statement best describes the difference in price of the DVD player at the two stores after discounts?

A The DVD player costs $10.20 less at Store B than at Store A.
B The DVD player costs $23.05 more at Store B than at Store A.
C The DVD player costs $10.20 more at Store B than at Store A.
D The DVD player costs $23.05 less at Store B than at Store A.

**Question 95**

Jessica is decorating a section of one wall of her kitchen with ceramic tiles. The area she is decorating is represented by the shaded area in the diagram below. She is using 3-inch square tiles. The tiles are sold in boxes of 100.

How many boxes of tiles will Jessica need to cover that section of the wall?

A 3
B 1
C 4
D 2

**Question 96**

A small company that manufactures staplers estimates that the cost per day in dollars of producing \( n \) staplers, is given by the formula \( c = 1.12n + 300 \), where $300 represents the fixed costs associated with operating the shop for a day. By about what percent would the cost of producing 250 staplers increase if the company’s fixed costs increased by 15%?

A 15%
B 8%
C 18%
D 6%

**Question 97**

Jessie and Philippe are on opposite ends of a road that is 5 miles long. Jessie is walking toward Philippe at 4 miles per hour, and Philippe is riding his bike toward Jessie at 6 miles per hour. In how many minutes will they meet each other?

A 120 min
B 50 min
C 30 min
D 2 min
Question 98
A publisher wants to include in an art history book a reproduction of an original painting that measures 2 feet wide by 3 feet tall. The publisher has a space for the picture that is 8 inches wide and 9 inches high. By what scale factor must the original painting be reduced if it is to be as large as possible, and still fit in the space available?

A $\frac{1}{7}$  
B $\frac{1}{4}$  
C 4  
D $\frac{1}{3}$  

Question 99
As a candle burns, its height decreases. Matt’s science class measured the height of a candle as it burned. They discovered that $h$, its height in inches, could be represented by the function $h = 12 - 0.1m$, where $m$ equals the number of minutes the candle burns.

Which of the following statements is not true?

A There is a linear relationship between the candle’s height and the number of minutes it burns.  
B The candle was 12 inches tall when it started burning.  
C The height of the candle is directly proportional to the number of minutes it burns.  
D The candle burned for at most 120 minutes.

Question 100
Which problem can be solved using the equation $3x + 40 = 100$?

A Carrie bought a pair of shoes for $40, a shirt, and a pair of pants that cost twice as much as the shirt. She spent a total of $100. What was the cost of the shirt?  
B The perimeter of a rectangle is 100 units. The length of the rectangle is 40 units more than three times the width. What is the width?  
C Alex deposited $100 at a 3% yearly interest rate. After how many years will he earn $40 in interest?  
D The student council spent $40 on cups with the school logo. The cups sell for $3 each. How many cups must the council sell to make a profit of $100?

Question 101
Joyce has been given two linear functions, $f(x) = 2(x - 3) + 7$ and $g(x) = \frac{1}{2}x + 7$. She wants to find the values of $x$ for which $f(x) \geq g(x)$.

Which of the following would be a reasonable strategy Joyce could use to solve the problem?

A Write the two functions in slope-intercept form and compare their slopes.  
B Write the two functions in slope-intercept form and compare their intercepts.  
C Solve the inequality $2x + 1 \geq \frac{1}{2}x + 7$.  
D Graph the two functions and see which graph has a positive slope.
Objective 10

Question 102
The table below shows the graphs of three related quadratic functions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = (x - 1)^2 )</td>
<td>![Graph A]</td>
</tr>
<tr>
<td>( y = (x - 3)^2 )</td>
<td>![Graph B]</td>
</tr>
<tr>
<td>( y = (x - 5)^2 )</td>
<td>![Graph C]</td>
</tr>
</tbody>
</table>

Which of the following is most likely the graph of \( y = (x - 2)^2 \)?

![Graph D]
Question 103

Look at the pattern of the cylinders below.

If the pattern continues, which of the following would be a correct step to find the volume of the next cylinder in this series?

A  Increase the diameter and height of the previous cylinder by a scale factor of 1.5.
B  Increase the diameter and height of the previous cylinder by a scale factor of \((1.5)^2\).
C  Increase the diameter and height of the previous cylinder by a scale factor of \((1.5)^3\).
D  Increase the diameter and height of the previous cylinder by a scale factor of \(\sqrt{1.5}\).

Question 104

Andrew is keeping a record of the number of people who visit his website each week. His site had twice as many visitors the second week as the first. The third week the site had 22 more visitors than the second week. The fourth week it had 2.5 times as many visitors as the first week. If \(x\) represents the number of visitors the site had during the first week, which expression could be used to find the mean number of visitors?

A  \(x + 2x + (2x + 22) + 2.5x\)
B  \(\frac{x + 2x + (2x + 22) + 2.5x}{4}\)
C  \(x + 2x + (x + 22) + 2.5x\)
D  \(\frac{x + 2x + (2x + 22) + 2.5(x + 22)}{4}\)
Objective 1

Question 1 (page 26)
A Incorrect. The total salary she is paid for a week depends on the number of hours she works, which is the independent quantity.
B Incorrect. Her hourly rate of pay, $5.50, is a constant.
C Correct. The total salary she is paid for a week is the dependent quantity. It depends on the number of hours she works, which is the independent quantity. Her hourly rate of pay, $5.50, is a constant.
D Incorrect. The number of days worked in a week is not a variable in this problem.

Question 2 (page 26)
C Correct. The number of pretzels in the box, p, is determined by the volume of the box, V. Since p depends on the value of V, it is the dependent variable.

Question 3 (page 26)
C Correct. The ordered pairs (1, 1) and (1, 16) both belong to the relationship represented in C. If this relationship were a function, each first coordinate would be paired with exactly one second coordinate. In this choice the first coordinate, 1, is paired with two different second coordinates, 1 and 16. Choice C is not a function.

Question 4 (page 26)
B Correct.
Check the ordered pairs in the table to see whether they satisfy the rule for the function.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Rule</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( f(x) = 2x^2 + 2x + 1 )</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>( f(0) = 2 \cdot 0^2 + 2 \cdot 0 + 1 )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( f(1) = 2 \cdot 1^2 + 2 \cdot 1 + 1 )</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>( f(2) = 2 \cdot 2^2 + 2 \cdot 2 + 1 )</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>( f(3) = 2 \cdot 3^2 + 2 \cdot 3 + 1 )</td>
<td>25</td>
</tr>
</tbody>
</table>

The ordered pairs in table B correctly represent the function \( f(x) = \frac{2}{5}x - 3 \).

Question 5 (page 27)
D Correct. Jane’s profit is equal to the difference between the amount of money she collects and her expenses. The variable \( x \) stands for the number of pets she cares for. The amount she collects for caring for \( x \) pets is \( 25x \). Her expenses are $210. The difference, \( 25x - 210 \), represents her profit. Therefore, the function \( f(x) = 25x - 210 \) best represents her net profit in terms of \( x \), the number of pets she cares for.

Question 6 (page 27)
A Correct. To find the correct inequality, first state the problem in words using the variable, the constants, and the relationship among them.

The current volume, 75 decibels, plus the amount by which the volume is increased, \( q \), must be less than 120 decibels, the level at which neighbors would complain.

Substitute the appropriate symbols for the quantities in an inequality: \( 75 + q < 120 \).

Question 7 (page 27)
B Correct.
Check the ordered pairs in the table to see whether they satisfy the rule for the function.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Rule</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( f(-10) = \frac{2}{5}(-10) - 3 )</td>
<td>-7</td>
</tr>
<tr>
<td></td>
<td>( f(-10) = -4 )</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>( f(-10) = -7 )</td>
<td>-3</td>
</tr>
<tr>
<td>10</td>
<td>( f(0) = \frac{2}{5}(0) - 3 )</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>( f(0) = 0 )</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>( f(0) = -3 )</td>
<td>-3</td>
</tr>
<tr>
<td>10</td>
<td>( f(10) = \frac{2}{5}(10) - 3 )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( f(10) = 4 )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( f(10) = 1 )</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>( f(20) = \frac{2}{5}(20) - 3 )</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>( f(20) = 8 )</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>( f(20) = 5 )</td>
<td>5</td>
</tr>
</tbody>
</table>

The function \( f(x) = 2x^2 + 2x + 1 \) correctly matches the dependent and independent quantities in the table.
**Question 8 (page 28)**

**C Correct.**

To verify that a graph represents a function, find the coordinates of several points on the graph and substitute them into the rule for the function.

For example, \((0, -9)\) is a point on the graph in choice C.

If \(x\) is replaced with 0 and \(y\) is replaced with \(-9\), is the functional rule satisfied? In other words, does \(f(x) = -9\) when \(x = 0\)?

\[ f(0) = x^2 - 9 \]

\[ f(0) = 0 - 9 \]

\[ f(0) = -9 \]

Yes, the point \((0, -9)\) is an ordered pair in the function \(f(x) = x^2 - 9\).

In the same way, check any other points on the graph. All the points on the graph in choice C satisfy the function \(f(x) = x^2 - 9\).

The graph in choice C correctly represents the function \(f(x) = x^2 - 9\).

**Question 9 (page 29)**

**A Incorrect.** The graph must start at 35 grams, not 0 grams, to account for the mass of the beaker.

**B Incorrect.** The mass of the beaker increases at a constant rate. A constant rate of increase is a linear function. The graph in choice B is not linear.

**C Correct.** The graph starts at 35 grams, which is the mass of the empty beaker. The mass increases at a constant rate. A constant rate of increase is a linear function. The graph increases and is a line. The graph ends at 50 grams of water added and a total mass of 85 grams. The graph in choice C best represents the combined mass of the beaker and the water as the amount of water in the beaker increases.

**D Incorrect.** The graph starts at 35 grams, which is correct. The graph ends at 10 grams of water added and a total mass of 85 grams. This is not the correct amount added to obtain a total mass of 85 grams. This graph cannot be correct.

**Question 10 (page 30)**

**D Correct.** The number of chocolate chip cookies sold is described in terms of the number of peanut butter cookies sold. Therefore, an ordered pair belonging to the function should list the number of peanut butter cookies sold as the \(x\)-coordinate and the number of chocolate chip cookies sold as the \(y\)-coordinate. Find several ordered pairs belonging to the function and then see which graph includes these ordered pairs.

The second coordinate should be twice the first coordinate. Ordered pairs such as \((1, 2), (2, 4),\) and \((5, 10)\) belong to this function. Only the graph in choice D contains ordered pairs that fit this pattern.

**Question 11 (page 31)**

**A Correct.** The class begins to make a profit when the accumulated sales exceed the expenses. To find this point, draw a dashed line extending horizontally through $1025 on the vertical axis.

Notice that the values on the vertical axis are in hundreds of dollars, so the dashed line representing $1025 should be just above the 10 mark on the vertical scale (since \(10 \times 100 = 1000\)). The line crosses the graph between Day 4 and Day 5. Thus, part of the sales on Day 4 and Day 5 were used to meet expenses, but the remaining sales that day were the profit.
Question 12 (page 31)

D  Correct. The horizontal scale of the graph represents temperature. Draw a vertical dashed line at 42°C to see where the line intersects the graph.

Read the corresponding value on the vertical axis, which is about 68 grams.

Question 13 (page 32)

C  Correct.

Evaluate each of the formulas for 1,450 ft² and compare the results.

<table>
<thead>
<tr>
<th>Community</th>
<th>Cost of Building a New Home</th>
<th>Evaluated for f = 1,450</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>c = 15,000 + 80f</td>
<td>c = 15,000 + 80(1,450)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c = 15,000 + 116,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c = 131,000</td>
</tr>
<tr>
<td>S</td>
<td>c = 25,000 + 75f</td>
<td>c = 25,000 + 75(1,450)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c = 25,000 + 108,750</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c = 133,750</td>
</tr>
<tr>
<td>T</td>
<td>c = 60,000 + 50f</td>
<td>c = 60,000 + 50(1,450)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c = 60,000 + 72,500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c = 132,500</td>
</tr>
<tr>
<td>V</td>
<td>c = 40,000 + 65f</td>
<td>c = 40,000 + 65(1,450)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c = 40,000 + 94,250</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c = 134,250</td>
</tr>
</tbody>
</table>

The least expensive community in which to build a 1,450 ft² home would be Community R.

Question 14 (page 59)

B  Correct. The graph is a parabola. Its parent function is the quadratic parent function, \( y = x^2 \), whose graph is also a parabola.

Question 15 (page 59)

B  Correct. The value of the account, \( v \), depends on the number of years the money is in the bank, \( t \). Therefore, \( v \) is the dependent variable. The range is the set of possible values for the dependent variable. The amount of money in the bank begins at $550 and increases each year thereafter. The minimum value for \( v \) is $550, but it has no upper limit. The range of the function is any value for \( v \) that is equal to or greater than $550, or \( v \geq 550 \).

Question 16 (page 59)

A  Incorrect. The speed would be very slow at the top of the track and would increase as the roller coaster approached the bottom. The speed would then decrease as the roller coaster climbed back up the track. The graph, however, decreases and then increases. The graph does not match the description of the roller coaster’s motion.

B  Incorrect. The price of a stock, the dependent quantity in choice B, decreases and then increases like the graph, but the stock price decreases to only half its original value, not to 0. The line of the graph falls to 0, so it does not match this description.

C  Correct. The bullet starts at a very high speed. It slows as it goes straight up until it reaches a maximum height, when its speed falls to 0. The bullet’s speed then increases as it falls back to Earth. The graph decreases and increases in the same fashion. The graph matches this description.

D  Incorrect. The race car must start from a speed of 0. Its speed would build to a maximum and stay near that speed for several laps. The speed of the race car would then go back to 0 for the fuel stop. The graph does not match this description.

Question 17 (page 60)

C  Correct. When there is a negative correlation between two quantities, one variable increases as
the other decreases. In this case, as time increases, the number of gallons of gasoline in the boat decreases. There is a negative correlation between the number of gallons of gasoline remaining in the boat and the number of hours that have passed.

**Question 18 (page 60)**

**A Correct.** The number of customers increases by 6 each day, and the amount of sales increases by $30. Continue the sequence until the amount of sales reaches $330. It will take 2 more days because $270 + 30 + 30 = 330$. The number of customers corresponding to this sales amount would be an increase of 6 per day for 2 days, or $30 + 6 + 6 = 42$ customers.

**Question 19 (page 61)**

**D Correct.** The volume of a rectangular prism (the refrigerator’s shape) is equal to its length times its width times its height. If $x$ represents the depth, or length, of the refrigerator, then $1.75x$ represents its width. The height of the refrigerator is 6 feet.

\[
V = lwh \\
V = x \cdot 1.75x \cdot 6 \\
V = (1.75 \cdot 6)(x \cdot x) \\
V = 10.5x^2
\]

**Question 20 (page 61)**

**C Correct.** Look for a pattern in the ordered pairs. The function appears to satisfy the rule that a term in the sequence is equal to twice its square, \( f(x) = 2x^2 \). See whether each term satisfies this rule.

<table>
<thead>
<tr>
<th>Position (n)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

Test the values in the table against the rule in choice C. For example, the rule in choice C says that the third term in the sequence should be \( 2n - 1 = 2 \cdot 3 - 1 \), or 5. When \( n = 3 \), the rule gives the correct term in the sequence. In the same way, the rule \( 2n - 1 \) predicts each of the other values in the sequence.

**Question 21 (page 61)**

**C Correct.** One way to find the relationship between the terms in a sequence and their position in the sequence is to build a table. Sequence 1, 3, 5, 7, 9, ...

<table>
<thead>
<tr>
<th>x</th>
<th>f(x) = 2x^2</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 (\frac{n}{2} \cdot 2 \cdot 1) 2 (\frac{n}{2} \cdot 2 \cdot 1) 2 = 2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8 (\frac{n}{2} \cdot 2 \cdot 2) 8 (\frac{n}{2} \cdot 2 \cdot 4) 8 = 8</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>18 (\frac{n}{2} \cdot 2 \cdot 3) 18 (\frac{n}{2} \cdot 2 \cdot 9) 18 = 18</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>32 (\frac{n}{2} \cdot 2 \cdot 4) 32 (\frac{n}{2} \cdot 2 \cdot 16) 32 = 32</td>
<td>32</td>
</tr>
</tbody>
</table>

Each ordered pair satisfies the rule \( f(x) = 2x^2 \).

**Question 22 (page 62)**

The correct answer is 6004. Use the functional rule, \( P(x) = \frac{1}{2}x - 2 \), to find the minimum number of loaves of bread Mr. Jones must sell to produce a profit of $3000.

\[
\frac{1}{2}x - 2 \geq 3000 \\
\frac{1}{2}x \geq 3002 \\
x \geq 6004
\]

Mr. Jones must sell at least 6004 loaves of bread to have a profit of at least $3000.
Question 23 (page 62)
C Correct. To find an equivalent expression, simplify the given expression by removing parentheses and combining like terms.

\[ 2x + 2(3x - 4) + 3(8x - 4) = \]
\[ 2x + (6x - 8) + (24x - 12) = \]
\[ (2x + 6x + 24x) + (-8 - 12) = \]
\[ 32x - 20 \]

Question 24 (page 89)
A Incorrect. The formula for the area of a circle, \( A = \pi r^2 \), involves a squared term, \( r^2 \). It is not a linear function.
B Correct. The formula for the perimeter of an equilateral triangle in terms of its side, \( s \), is \( P = 3s \). All variables are to the first power, so this is a linear function.
C Incorrect. The surface area of a cube with side length \( s \) is given by the formula \( A = 6s^2 \). The function involves a squared term, \( s^2 \). It is not a linear function.
D Incorrect. The formula for the volume of a cylinder with radius \( r \) and height \( h \) is \( V = \pi r^2 h \). The function involves the product of two independent variables, \( r \) and \( h \), and includes a variable raised to the second power. It is not a linear function.

Question 25 (page 89)
B Correct.
One way to match the equation to its graph is to write the equation \( 2y - x = 10 \) in slope-intercept form, \( y = mx + b \).

\[ 2y - x = 10 \]
\[ 2y = x + 10 \]
\[ y = \frac{1}{2} x + 5 \]

In this form the slope, \( m \), is \( \frac{1}{2} \), and the y-intercept, \( b \), is 5. Only the graph in choice B has a slope of \( \frac{1}{2} \) and a y-intercept of 5.

Question 26 (page 90)
A Incorrect. The rate of change is the slope of the line graphed.

\[ \text{Slope} = \frac{-3 - (-1)}{4 - 1} = \frac{-2}{3} \]

The graph represents a rate of change of \( -\frac{2}{3} \).
B Incorrect. To find the slope of the line, write the equation in slope-intercept form, \( y = mx + b \).

The equation becomes \( y = -\frac{2}{3}x + 4 \), so the slope is \( -\frac{2}{3} \).

The equation represents a rate of change of \( -\frac{2}{3} \).
C Correct. To find the rate of change represented in the table, compare the difference between any two \( y \)-values to the corresponding difference in \( x \)-values. For example, use the two points (1, 4) and (-3, 10).

\[ \text{Rate of change} = \frac{10 - 4}{-3 - 1} = \frac{6}{-4} = -\frac{3}{2} \]

The table does not represent a rate of change of \( -\frac{2}{3} \).
D Incorrect. The equation is already in slope-intercept form, \( y = mx + b \). The value of \( m \) is \( -\frac{2}{3} \).

Therefore, the slope is \( -\frac{2}{3} \). The equation represents a rate of change of \( -\frac{2}{3} \).

Question 27 (page 90)
B Correct. To find the maximum number of grams of potatoes, look at the point on the graph where the number of grams of beef equals zero, the \( x \)-intercept. The graph crosses the \( x \)-axis at the point (450, 0). Therefore, the maximum number of grams of potatoes that you could eat to obtain 500 calories is 450 grams.

Question 28 (page 91)
C Correct. The \( y \)-intercept is (0, 200). The \( y \)-coordinate, 200, represents 200 units on the \( y \)-axis. The units on the \( y \)-axis are thousands of dollars. The company's initial assets are \( 200 \cdot $1,000 = $200,000 \).

The slope, or rate of growth, is 50 units in the \( y \)-direction for every unit in the \( x \)-direction. Thus, the slope is \( \frac{50}{1} \), or 50. The \( y \)-axis units are thousands of dollars, and the \( x \)-axis units are years. The expected growth rate is $50,000 per year.
Question 29 (page 91)
B Correct.
To find the slope of the graph of the given equation, \(8x + 2y = 10\), write it in slope-intercept form, \(y = mx + b\). Solve for \(y\) by subtracting \(8x\) from both sides of the equation and then dividing both sides by 2. The transformed equation is \(y = -4x + 5\). This form shows that the value of \(m\) is \(-4\). The slope of the given equation is \(-4\).

Identify the answer choice whose graph has a slope equal to \(-4\) and a \(y\)-intercept equal to 8. To find the slope and \(y\)-intercept of the line in choice B, solve for \(y\). The equation \(4x + y = 8\) is equivalent to the equation \(y = -4x + 8\). This line has a slope of \(-4\) and a \(y\)-intercept of 8. It is the only answer choice that meets both requirements.

Question 30 (page 91)
A Correct.
Both graphs are lines. To determine their relationship, compare the slope of each graph. The first equation, \(y = \frac{3}{4}x - 4\), is written in slope-intercept form, \(y = mx + b\). Its slope is the value of \(m\), or \(\frac{3}{4}\). The second equation has a slope of 1. Since \(1 > \frac{3}{4}\), the slope of the second line is greater than that of the first line. Therefore, the second line is steeper than the first line.

Question 31 (page 91)
D Correct.
Use the slope formula to find the slope of the line that passes through the two points \((2, -5)\) and \((4, 3)\).
\[
m = \frac{-5 - 3}{2 - 4} = \frac{-8}{-2} = 4
\]
The slope of the line is 4.
Substitute 4 for \(m\) and the coordinates of the point \((2, -5)\) for \(x\) and \(y\) in the slope-intercept form of the equation of a line.
\[
y = mx + b
\]
\[
-5 = 4 \cdot 2 + b
\]
\[
-5 = 8 + b
\]
\[
b = -13
\]
Substituting \(m = 4\) and \(b = -13\) into the slope-intercept form of the equation of a line results in \(y = 4x - 13\). The equation \(y = 4x - 13\) is equivalent to the equation in choice D.
\[
y = \frac{4x - 13}{-4x + y = -13}
\]

Question 32 (page 91)
D Correct.
One way to find the two intercepts is to write the equation in standard form, \(Ax + By = C\). Then replace \(x\) with 0 to determine the \(y\)-intercept and replace \(y\) with 0 to determine the \(x\)-intercept.
The equation \(2x = 9 - 3y\) in standard form is \(2x + 3y = 9\). Let \(y = 0\).
\[
2x + 3 \cdot 0 = 9
\]
\[
2x = 9
\]
\[
x = \frac{9}{2}
\]
The \(x\)-intercept is \(\frac{9}{2}\). The coordinates of the \(x\)-intercept are \(\left(\frac{9}{2}, 0\right)\).
In the same way, let \(x = 0\).
\[
2 \cdot 0 + 3y = 9
\]
\[
3y = 9
\]
\[
y = 3
\]
The \(y\)-intercept is 3. The coordinates of the \(y\)-intercept are \((0, 3)\).

Question 33 (page 92)
A Correct. The slopes of the lines represent their rate of change, the price per cookie. The slopes of the two lines are equal, so the price per additional cookie remained the same. The first graph contains the point \((12, 6)\), and the second graph contains the point \((12, 7)\). The cost of the first dozen cookies increased from $6 to $7.

Question 34 (page 92)
A Correct. The number of miles Sammie walks is directly proportional to the number of minutes she walks. Write a direct-proportion equation.
\[
y = kx
\]
In this equation, \( y \) is the number of miles and \( x \) is the number of minutes. She walks 3 miles in 45 minutes. Substitute and solve for \( k \), the proportionality constant.

\[
y = kx \\
3 = k \cdot 45 \\
\frac{3}{45} = \frac{k \cdot 45}{45} \\
k = \frac{1}{15}
\]

The constant of proportionality is \( \frac{1}{15} \).

The equation describing this situation is \( y = \frac{1}{15}x \). Find the number of miles walked in 2.5 hours. Convert 2.5 hours to minutes so that the units are the same.

\[2.5 \text{ hours} \cdot 60 \frac{\text{minutes}}{\text{hour}} = 150 \text{ minutes}.
\]

Substitute 150 for \( x \) in the equation \( y = \frac{1}{15}x \).

Solve the equation for \( y \).

\[
y = \frac{1}{15} \cdot 150 \\
y = 10
\]

At this rate Sammie would walk 10 miles in 2.5 hours.

**Objective 4**

**Question 35 (page 106)**

C Correct. This is a problem about the perimeter of a rectangle. The width of the rectangle is given as \( w \). The length will be five feet more than the width, so the length can be represented by \( w + 5 \). The formula for the perimeter of a rectangle is \( P = 2w + 2l \). The perimeter of the rectangle can be represented by the equation \( P = 2w + 2(w + 5) \).

She can afford 150 feet of fencing. The perimeter must be less than or equal to 150 feet. Use the symbol \( \leq \) to write an inequality expressing this relationship.

\[
P \leq 150 \\
2w + 2(w + 5) \leq 150
\]

**Question 36 (page 106)**

C Correct. Let \( m \) represent the amount of money she spent on movie rentals, \( c \) the cost of her car repairs, and \( l \) the cost of her lunches. Since Sharon spent five times as much on car repairs, \( c \), as she did on movie rentals, \( m \), you can write the equation \( c = 5m \). Since she spent \$4 less on lunches, \( l \), than on movie rentals, \( m \), you can write the equation \( l = m - 4 \).

The sum of these expenses is \$80, so \( m + c + l = 80 \).

Substitute the expressions in terms of \( m \) for \( c \) and \( l \) and solve for \( m \).

\[
m + c + l = 80 \\
7m - 4 = 80 \\
m = 12
\]

Sharon spent \$12 on movie rentals. The question asks how much her car repairs cost. Since \( c = 5m \), her car expenses were \( 5 \cdot 12 = \$60 \).

**Question 37 (page 106)**

A Correct. Represent the first number with \( x \) and the second number with \( y \).

If the sum of the two numbers is 59, then \( x + y = 59 \).

If the difference between 2 times the first number, \( x \), and 6 times the second number, \( y \), is \(-34\), then \( 2x - 6y = -34 \).

Solve the system of equations.

\[
x + y = 59 \\
2x - 6y = -34
\]

Addition method: Multiply the first equation by 6 to obtain two terms with opposite coefficients, 6 and \(-6\).

\[
6x + 6y = 354 \\
2x - 6y = -34
\]

Add the two equations to obtain one equation with one unknown.

\[
8x = 320 \\
x = 40
\]

If \( x = 40 \), then substitute 40 into the first equation to find \( y \).

\[
x + y = 59 \\
40 + y = 59 \\
y = 19
\]

The first number is 40, and the second number is 19.

Substitution method: Solve the first equation for \( y \).

\[
y = 59 - x
\]

Substitute \((59 - x)\) for \( y \) in the second equation.
2x - 6y = -34
2x - 6(59 - x) = -34
2x - 354 + 6x = -34
8x - 354 = -34
8x = 320
x = 40

If x = 40, then substitute 40 into the first equation, x + y = 59, to find y.

y = 19

The first number is 40, and the second number is 19.

Question 39 (page 107)

D Correct. Let \( D \) be the width of the kennel. Ira must fence two widths plus the length of the garage. The total number of feet of fencing Ira will use is \( w + w + 20 = 2w + 20 \). The total cost of materials can be represented by the number of feet of fencing times the cost per foot of the materials, \( 4(2w + 20) \). If Ira has at most $120 to spend on the project, then the inequality \( 4(2w + 20) \leq 120 \) can be used to solve the problem.

\[
4(2w + 20) \leq 120
\]
\[
8w + 80 = 120
\]
\[
8w \leq 40
\]
\[
w \leq 5
\]

The kennel can be at most 5 feet wide.

Question 40 (page 107)

C Correct. Write the two equations in slope-intercept form, \( y = mx + b \).

\[
3x - 2y = 14 \quad \text{and} \quad 6x - 2y = 32
\]
\[
\begin{align*}
-2y &= -3x + 14 \\
-2y &= -6x + 32
\end{align*}
\]
\[
\begin{align*}
\frac{-2y}{-2} &= \frac{-3x + 14}{-2} \\
\frac{-2y}{-2} &= \frac{-6x + 32}{-2}
\end{align*}
\]
\[
y = \frac{3}{2}x - 7 \quad \text{and} \quad y = 3x - 16
\]

The slope of the graph of the first equation is \( \frac{3}{2} \), and the slope of the graph of the second equation is 3. The \( \text{y-intercept of the first equation is } (0, -7) \), and the \( \text{y-intercept of the second equation is } (0, -16) \). Since the slopes are different, the graphs will be a pair of intersecting lines. The single point at which the lines intersect represents the one solution.

Question 41 (page 107)

A Correct. The cost of an adult ticket, \( a \), is twice as much as the cost of a child ticket, \( c \). Therefore, \( a = 2c \). Sandra bought 3 adult tickets and 5 child tickets. The total cost of the tickets, $48.40, is equal to 3 times \( a \), the cost of an adult ticket, plus 5 times \( c \), the cost of a child ticket. So

\[
3a + 5c = 48.40
\]

To find the cost of each ticket, solve the following system of equations.

\[
a = 2c
\]
\[
3a + 5c = 48.40
\]

Use the substitution method because the first equation is already solved for \( a \).

Substitute the expression \( 2c \) for \( a \) in the second equation and solve.

\[
3(2c) + 5c = 48.40
\]
\[
6c + 5c = 48.40
\]
\[
11c = 48.40
\]
\[
c = 4.40
\]

Since \( a = 2c \), the cost of an adult ticket, \( a \), is

\[
2($4.40) = $8.80
\]

The cost of an adult ticket is $8.80, and the cost of a child ticket is $4.40.

Question 42 (page 107)

A Correct. The store’s profit is given in the equation

\[
p = 0.25(s - 3000)
\]

Substitute the greatest and least projected values for the monthly sales, \( s \), to find the expected range of values for the profit, \( p \).

Solve both inequalities to find the possible range of values for \( p \).

\[
p = 0.25(s - 3000)
\]
\[
p \geq 0.25(5000 - 3000) \quad p \leq 0.25(7000 - 3000)
\]
\[
p \geq 0.25(2000) \quad p \leq 0.25(4000)
\]
\[
p \geq 500 \quad p \leq 1000
\]

This means that \( p \geq 500 \) and \( p \leq 1000 \), or

\[
500 \leq p \leq 1000
\]
Question 43 (page 108)
C Correct. The solution can be represented by graphing the inequality and identifying points in the shaded region.

Only the point (20, 50) is not in the shaded region of the graph.

If Brent sold 20 cans of popcorn, he would have raised 20 \cdot $5 = $100. And 50 candy bars would have raised 50 \cdot $2 = $100. He would have raised only $200 in all, not more than $300.

Question 44 (page 108)
D Correct. In general, the total cost of staying at the resort is equal to the cost of a night, $n$, times the number of nights, plus the cost of a meal, $m$, times the number of meals.

If a guest stays 2 nights and has 5 meals, it costs $395. Represented by an equation in terms of $n$ and $m$, this is $2n + 5m = 395$.

If a guest stays 5 nights and has 11 meals, it costs $959. Represented by an equation in terms of $n$ and $m$, this is $5n + 11m = 959$.

Question 45 (page 108)
D Correct. The total number of watermelons and cantaloupes bought is 13.

The number of watermelons, $w$, plus the number of cantaloupes, $c$, is 13.

$$w + c = 13$$

The total amount spent on watermelons is equal to $2$ times $w$, the number of watermelons, or $2w$.

The total amount spent on cantaloupes is equal to $1$ times $c$, the number of cantaloupes, or $1c$.

The total amount spent on watermelons and cantaloupes, $20$, is equal to the sum of these two amounts.

$$2w + c = 20$$

Objective 5

Question 46 (page 130)
C Correct. Multiplying the coefficient of $x^2$ by 2 in the equation $y = -2x^2$ changes the equation to $y = -4x^2$. Compare the absolute values of the coefficients. The absolute value of the coefficient in the new equation is larger than in the old equation. This increase causes the graph to appear narrower.

Question 47 (page 130)
B Correct. The value of the parameter $a$ in the function $y = \frac{1}{2}x^2$ is $\frac{1}{2}$. The value of $a$ in the function $y = -\frac{1}{2}x^2$ is $-\frac{1}{2}$. Changing the sign of $a$ causes the graph to be reflected across the $x$-axis. The two graphs are congruent.
Question 48 (page 130)

D Correct.
The vertex of the graph of \( y = x^2 + 6 \) is 6 units above the origin, and the vertex of the graph of \( y = x^2 - 1 \) is 1 unit below the origin. The vertex of \( y = x^2 + 6 \) is 7 units above the vertex of \( y = x^2 - 1 \).

Question 49 (page 131)

D Correct. As the selling price increases, the profit increases until it reaches a maximum; after the maximum profit is reached, the profit decreases as the selling price continues to increase. A selling price that is neither too high nor too low produces the best profit.

Question 50 (page 132)

A Incorrect. The horizontal axis represents time, not distance.

B Incorrect. The horizontal axis represents time, not distance. The interpretation of the graph as the path of the stone is not correct. In this instance, the path of the stone is not a curving path, but a vertical line.

C Correct. The horizontal axis of the graph represents time in seconds from the time the stone is dropped. The stone's distance from the ground is 0 feet when the graph crosses the horizontal axis. The graph crosses this axis between 5 and 6, so the stone hits the ground between 5 and 6 seconds after it is dropped.

D Incorrect. The stone's distance from the ground does not decrease at a constant rate. If the distance decreased at a constant rate, the graph would be linear, not curved.

Question 51 (page 133)

B Correct.
The roots of a function are the values of the independent variable, \( x \), for which the dependent variable, \( y \), is 0. The roots of the function can be seen by examining the table. There are two ordered pairs in the table that have 0 values for \( y \): (0, 0) and (4, 0). The \( x \)-values for these ordered pairs are 0 and 4.

Question 52 (page 133)

A Correct.
Write the equation \( 2x^2 + 3x = 9 \) in standard form, \( 2x^2 + 3x - 9 = 0 \). The quadratic expression \( 2x^2 + 3x - 9 \) is factorable. Factor the expression and rewrite the equation. One way to solve this equation is to set each factor equal to 0.

\[
(2x - 3)(x + 3) = 0
\]

\[
x = \frac{3}{2} \quad \text{and} \quad x = -3
\]

The solutions are \( x = \frac{3}{2} \) and \( x = -3 \).

Question 53 (page 133)

D Correct.
In the equation \( x^2 - 4x - 5 = 0 \), replace \( x \) first with \( -1 \) and then with \( 5 \) to see whether the equation is true.

Substitute \( x = -1 \)

\[
x^2 - 4x - 5 = 0
\]

\[
(-1)^2 - 4 \cdot (-1) - 5 = 0
\]

\[
1 + 4 - 5 = 0
\]

\[
0 = 0
\]

Substitute \( x = 5 \)

\[
x^2 - 4x - 5 = 0
\]

\[
(5)^2 - 4 \cdot 5 - 5 = 0
\]

\[
25 - 20 - 5 = 0
\]

\[
0 = 0
\]

Both replacements make this equation true. Only the equation in choice D has \( -1 \) and \( 5 \) as solutions.
MATHEMATICS

Question 54 (page 134)
A Correct. Use the quadratic formula to determine the roots of the equation. Substitute $a = 1$, $b = -4$, and $c = 2$ into the quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The following result is obtained.

$$\frac{(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$

$$\frac{4 \pm \sqrt{16 - 8}}{2}$$

$$\frac{4 \pm \sqrt{8}}{2}$$

Since $\sqrt{8} \approx 2.8$, substitute this value in the formula.

$$\frac{4 \pm 2.8}{2} \approx \frac{7.8}{2} = 3.9$$

$$\frac{4 - 2.8}{2} \approx 0.6$$

The roots of the equation are approximately 3.4 and 0.6, with 0.6 being the smaller root. The number 0.6 lies between the integers 0 and 1.

Question 55 (page 134)
D Correct. Substitute the given expression for the radius into the formula for volume of a sphere.

$$V = \frac{4}{3}\pi(3x^2y)^3$$

Following the order of operations, first simplify the expression for the radius cubed, $(3x^2y)^3$. When raising a term with an exponent to a power, multiply the exponents.

$$V = \frac{4}{3}(3x^2y)^3 \pi$$

$$V = \frac{4}{3} \cdot 3^3x^6y^3 \cdot \pi$$

$$V = \frac{4}{3} \cdot 27x^6y^3 \cdot \pi$$

$$V = 36x^6y^3\pi$$

Question 56 (page 147)
C Correct. Use the graph to find the coordinates of the triangle: $A (0, -2)$, $B (2, 2)$, and $C (3, -2)$. The length of side $AC$ is $3 - 0 = 3$, and the length of side $A'C'$ is $6 - 0 = 6$. The ratio of these lengths is $\frac{3}{6} = \frac{1}{2}$, which is the given dilation factor. Only the triangle in choice C has its side lengths in a $2:1$ ratio to those of the original triangle.

Question 57 (page 148)
A Correct. The side lengths of the original rectangle and the dilated rectangle are as follows:

$\frac{RS}{TV} = 3$, $\frac{ST}{RV} = 6$, $\frac{R'S'}{T'V'} = 1$, and $\frac{S'T'}{R'V'} = 2$.

To find the scale factor, choose one pair of corresponding sides.

Find the ratio of the side length of the dilated figure to that of the original figure.

$$\frac{R'V'}{RV} = \frac{2}{6} = \frac{1}{3}$$

The side length of the dilated figure is $\frac{1}{3}$ the side length of the original figure. The scale factor is $\frac{1}{3}$.

Question 58 (page 148)
A Incorrect. If one triangle is a dilation of the other, then the two triangles are similar. The lengths of the corresponding sides of similar triangles must be in equal ratios.

$$\frac{1}{1} = 1 \quad \frac{1.5}{\frac{3}{2}} = \frac{1}{\frac{3}{4}} = \frac{2}{4} = \frac{1}{2}$$

The ratios are not all equal.

B Incorrect. If one triangle is a dilation of the other, then the two triangles are similar. The lengths of the corresponding sides of similar triangles must be in equal ratios.

$$\frac{1}{2} = \frac{1}{2} \quad \frac{1.5}{\frac{3}{4}} = \frac{2}{6} = \frac{1}{3}$$

The ratios are not all equal.

C Incorrect. If one triangle is a dilation of the other, then the two triangles are similar. The lengths of the corresponding sides of similar triangles must be in equal ratios.

$$\frac{1}{4} = \frac{1}{4} \quad \frac{1.5}{\frac{3}{4}} = \frac{2}{5} = \frac{1}{5}$$

The ratios are not all equal.

D Correct. If one triangle is a dilation of the other, then the two triangles are similar. The lengths of the corresponding sides of similar triangles must be in equal ratios. Only the side lengths in choice D form a proportion with the given sides.

$$\frac{1}{4} = \frac{1}{4} \quad \frac{1.5}{\frac{3}{4}} = \frac{3}{12} = \frac{1}{4} \quad \frac{2}{8} = \frac{1}{4}$$
Question 59 (page 148)

A Correct. The point (5, 6) in the given pentagon is 5 units to the right of the y-axis. The point (−5, 6) is 5 units to the left of the y-axis. The point (−5, 6) is the reflection of the point (5, 6) across the y-axis. The point (4, 1) in the given pentagon is 4 units to the right of the y-axis. The point (−4, 1) is 4 units to the left of the y-axis. The point (−4, 1) is the reflection of the point (4, 1) across the y-axis. The point (3, 5) in the given pentagon is 3 units to the right of the y-axis. The point (−3, 5) is 3 units to the left of the y-axis. The point (−3, 5) is the reflection of the point (3, 5) across the y-axis.

Question 60 (page 148)

D Correct. After a translation of 2 units to the right and 3 units down, 2 is added to the x-coordinate of each point, and 3 is subtracted from the y-coordinate of each point.

Point R (1, 1) of the original quadrilateral has coordinates of x = 1 and y = 1. Under a translation of 2 units to the right, its x-coordinate will be (1 + 2) = 3. Under a translation of 3 units down, its y-coordinate will be (1 − 3) = −2.

Point R’ will have coordinates (3, −2).

Point T (5, 8) of the original quadrilateral has coordinates of x = 5 and y = 8. Under a translation of 2 units to the right, its x-coordinate will be (5 + 2) = 7. Under a translation of 3 units down, its y-coordinate will be (8 − 3) = 5.

Point T’ will have coordinates (7, 5).

Question 61 (page 149)

C Correct. The scale factor of the dilation is greater than 1. The figure will be enlarged.

The ratios of the lengths of corresponding sides should be \( \frac{2}{1} \).

Compare sides LM and L’M’ since they are vertical.

\[ L (3, 4), M (3, 6), L' (6, 8), M' (6, 12) \]

Segment LM is vertical and has a length of 6 − 4 = 2.

Segment L’M’ is vertical and has a length of 12 − 8 = 4.

The side lengths are in the ratio of \( \frac{2}{1} \). Only choice C has all the corresponding side lengths in this ratio.

Question 62 (page 149)

D Correct. The midpoint of a line segment is found by averaging the x-coordinates and averaging the y-coordinates.

\[ M = \left( \frac{-3}{2} + \frac{3}{2}, \frac{-7}{2} + \frac{-5}{2} \right) = \left( 0, \frac{-12}{2} \right) = (0, -6) \]

Question 63 (page 150)

A Incorrect. The coordinates of point R are (3, −2). The x-coordinate of point R is 3. Since \( 3 < \frac{3}{2} \), point R does not meet the requirement that \( -\frac{7}{2} < x < \frac{3}{2} \).

B Incorrect. The coordinates of point S are (−4, −3). The x-coordinate of point S is −4. Since \( -4 > -\frac{7}{2} \), point S does not meet the requirement that \( -\frac{7}{2} < x < \frac{3}{2} \).

C Correct. The coordinates of point T are (−2, 3). The x-coordinate of point T is −2.

\[ -2 > -\frac{7}{2} \text{ and } -2 < \frac{3}{2} \]

Thus, \( -\frac{7}{2} < x < \frac{3}{2} \). Only the x-coordinate of point T satisfies both of the given inequalities.

D Incorrect. The coordinates of point U are (4, 2). The x-coordinate of point U is 4. Since \( 4 > \frac{3}{2} \), point U does not meet the requirement that \( -\frac{7}{2} < x < \frac{3}{2} \).
Mathematics Answer Key

Question 64 (page 150)
D Correct. To translate a point 2 units to the right, add 2 to its x-coordinate. The x-coordinate of the point \((m, 2n)\) is \(m\). The x-coordinate of the translated point is 2 larger than \(m\), or \(m + 2\). The y-coordinate of the point is unaffected by a translation 2 units to the right. The y-coordinate of the point remains \(2n\). The coordinates of the translated point are \((m + 2, 2n)\).

Question 65 (page 150)
C Correct. The x-coordinate of the given point is \(\frac{8}{3} = \frac{2}{5}\). The point should be between 2 and 3 on the x-axis. The x-coordinates of points \(M\) and \(N\) are between 2 and 3. The y-coordinate of the given point is \(-\frac{9}{5} = -1\frac{4}{5}\). The point should be between \(-1\) and \(-2\) on the y-axis. The y-coordinates of points \(L\) and \(N\) are between \(-1\) and \(-2\). Only point \(N\) has both the correct x- and y-coordinates.

Objective 7

Question 66 (page 158)
A Incorrect. This is the top view of the object.
B Incorrect. This is the front view of the object.
C Correct. This is not a top, front, or side view of the object.
D Incorrect. This is the right-side view of the object.

Question 67 (page 159)
A Incorrect. This is the front view of the object.
B Incorrect. This is the right-side view of the object.
C Correct. This is the top view of the object.
D Incorrect. This view looks somewhat like the top view of the object, but it does not show enough squares.

Question 68 (page 159)
D Correct.

Use the formula for the area of a rectangle to find the areas of the two panels she painted.

\[
10 \text{ ft} \cdot 20 \text{ ft} = 200 \text{ ft}^2 \\
4 \text{ ft} \cdot 15 \text{ ft} = 60 \text{ ft}^2
\]

Find their sum.

\[
200 + 60 = 260 \text{ ft}^2
\]

Then subtract their sum from 400 to find the number of square feet that the paint left in the can will cover.

\[
400 \text{ ft}^2 - 260 \text{ ft}^2 = 140 \text{ ft}^2
\]

The remaining paint will cover at most 140 \(\text{ft}^2\). Only the rectangle in answer choice D has an area that is less than or equal to 140 \(\text{ft}^2\) because 10 \(\cdot\) 12 = 120, and 120 \(<\) 140.

Question 69 (page 160)
C Correct.

If the three lengths are to form the sides of a right triangle, then they must satisfy the Pythagorean Theorem, \(a^2 + b^2 = c^2\).

The greatest number in the set is 16. The hypotenuse of a right triangle is the longest side. Let \(c = 16\). The lesser numbers are 8 and 15. Let \(a = 8\) and \(b = 15\).

Does \(8^2 + 15^2 = 16^2\)?

No, because 64 \(\neq\) 256.

Does 64 + 225 = 256?

No, because 64 + 225 = 289. Since 289 \(\neq\) 256, the set of numbers 8, 15, and 16 could not be the sides of a right triangle.

Question 70 (page 160)

The correct answer is 10. Use a proportion to find the answer.

\[
\frac{18 \text{ ft}}{4 \text{ inches}} = \frac{45 \text{ ft}}{x \text{ inches}}
\]

18 = 45
4 = \(x\)

18x = 4 \cdot 45
18x = 180
\(x\) = 10

The longer side of the scale drawing is 10 inches.
Question 71 (page 161)
B Correct.
Since the tiles are in square inches, find all areas in square inches.
The area of the countertop is its length times its width.
\[ A = lw \]
\[ A = 5 \text{ ft} \times 2 \text{ in.} \times 2 \text{ ft} \times 2 \text{ in.} \]
\[ A = 64 \text{ in.} \times 26 \text{ in.} \]
\[ A = 1664 \text{ in.}^2 \]
The area of the circular sink equals \( \pi \) times its radius squared.
\[ A = \pi r^2 \]
\[ A = \pi \times 8^2 \]
\[ A = 64\pi \]
\[ A \approx 201 \text{ in.}^2 \]
The area of the countertop, not including the sink, is equal to the area of the rectangular top minus the area of the circular sink.
\[ 1664 - 201 = 1463 \text{ in.}^2 \]
Each tile is 2 in. by 2 in., so it has an area of 4 in. \(^2\).
Divide 1463 by 4 to find the number of tiles needed.
\[ 1463 \div 4 = 365.75 \]
Ed will need at least 366 tiles.

Question 72 (page 161)
B Correct. The set of squares in answer choice B shows that the triangle is a right triangle. The largest square has a side length of 5 units and an area of 25 square units. The other two squares have side lengths of 3 units and 4 units. They have areas of 9 square units and 16 square units, respectively.

If 25 = 9 + 16, then \( 5^2 = 3^2 + 4^2 \). This triangle shows that the sum of the squares of the legs equals the square of the hypotenuse. This model demonstrates the Pythagorean Theorem.

Question 73 (page 180)
B Correct.
To calculate the total surface area using a net, add the areas of all the surfaces.

The rectangle forms the curved surface of the cylinder. The length of the rectangle is equal to the circumference of the circular base.
\[ C = \pi d \]
\[ C = \pi \times 5 \]
\[ C \approx 15.7 \]
The rectangle has a length of 15.7 m and a width of 2.8 m. The area of the rectangle is \( 15.7 \times 2.8 = 43.96 \text{ m}^2 \).
The cylinder has two circular surfaces. Use the formula for the area of a circle. Each circle has a diameter of 5 m. The radius is half the diameter. The radius is \( 5 \div 2 = 2.5 \text{ m.} \) Use 2.5 for \( r \).
\[ A = \pi r^2 \]
\[ A = \pi \times (2.5)^2 \approx 19.635 \text{ m}^2 \]
The area of one circular surface is about 19.635 \( \text{ m}^2 \).
To find the surface area, add the areas of all the surfaces.
\[ S \approx 43.96 + 2 \times 19.635 \approx 83.23 \]
Rounding to the nearest square meter, the surface area of the cylinder is 83 \( \text{ m}^2 \).

Question 74 (page 180)
C Correct. The surface area of a prism is the sum of the areas of its surfaces.
To find the area of the triangular surfaces, measure the length of the base of the triangle and its height. The length of the base is 3 cm, and the height is 2.6 cm.

Use the formula for the area of a triangle.
\[ A = \frac{1}{2} bh \]
\[ A = \frac{1}{2} \times 3 \times 2.6 \]
\[ A = 3.9 \text{ cm}^2 \]
The area of each triangular surface is 3.9 \( \text{ cm}^2 \).
Find the area of the rectangular surfaces. Measure the length and width of one of the rectangles. The length is 4.4 cm, and the width is 3 cm. The area of each rectangular surface is \( 3 \times 4.4 \), or 13.2 \( \text{ cm}^2 \).
Find the sum of the areas of all the surfaces to find the total surface area of the prism. The prism has 2 triangular surfaces and 3 rectangular surfaces.
\[ S = 2 \times 3.9 + 3 \times 13.2 = 7.8 + 39.6 = 47.4 \text{ cm}^2 \]
Choice C, 47 \( \text{ cm}^2 \), is closest to this value.
Question 75 (page 181)

C  Correct. The volume of the figure is equal to the sum of the volumes of the cone and the cylinder. The base of a cone is a circle. Use the formula \( A = \pi r^2 \) to find \( B \), the area of the circular base. The radius of the circle measures 3.1 cm.

\[ B = \pi (3.1)^2 \]

The height, \( h \), of the cone is the perpendicular distance from the top of the cone to its base. The height measures 7.6 cm. Substitute these values into the formula for the volume of a cone.

\[ V = \frac{1}{3}Bh \]

\[ V = \frac{1}{3}\pi (3.1)^2 (7.6) \]

The height of the cylinder is 4 cm. Its base is a circle the same size as the cone’s base. The cylinder’s base also has an area of \( \pi (3.1)^2 \). Use the formula for the volume of a cylinder.

\[ V = Bh \]

\[ V = \pi (3.1)^2 (4) \]

The equation \( V = \frac{1}{3}\pi (3.1)^2 (7.6) + \pi (3.1)^2 (4) \) could be used to find the volume of the figure to the nearest cubic centimeter.

Question 76 (page 181)

B  Correct. The passageway is a rectangular prism. Calculate the volume of the prism.

\[ V = Bh = (lw)h \]

\[ V = (6 \cdot 4) \cdot 3 \]

\[ V = 72 \text{ ft}^3 \]

The part of the pipe that is inside the passageway is a cylinder that is 6 feet long, \( h = 6 \) ft. The radius, \( r \), of the cylinder is the diameter divided by 2.

\[ r = 30 \div 2 = 15 \text{ in.} \]

Convert the radius to feet; divide 15 by 12.

\[ r = 15 \div 12 = 1.25 \text{ feet} \]

Calculate the volume of the cylinder.

\[ V = Bh = \pi r^2 h \]

\[ V = \pi \cdot (1.25)^2 \cdot 6 \]

\[ V \approx 29.45 \text{ ft}^3 \]

Subtract the volume of the pipe from the volume of the passageway to find the volume of insulating material needed to fill the space around the pipe.

\[ 72 - 29.45 = 42.55 \approx 43 \text{ ft}^3 \]

To the nearest cubic foot, the volume of the space to be filled with insulating material is 43 cubic feet.

Question 77 (page 182)

B  Correct. On her walk to the scenic overlook, Jillian follows the sidewalk. She walks \( 300 + 475 = 775 \) ft. Compare this distance to the shortcut. Her journey along the sidewalk and her shortcut form a right triangle. The shortcut is the hypotenuse, \( c \), of the right triangle. Use the Pythagorean Theorem to calculate the length of the shortcut.

\[ c = \sqrt{300^2 + 475^2} = \sqrt{90,000 + 225,625} = \sqrt{315,625} \approx 561.81 \]

Subtract the length of the shortcut from the distance walked on the sidewalk to determine how much shorter the trip back to the parking lot is.

\[ 775 - 561.81 = 213.19 \text{ ft} \]

To the nearest whole foot, the walk back to the parking lot is 213 feet shorter.

Question 78 (page 182)

D  Correct. The triangles are similar, so the lengths of the corresponding sides are proportional. Write a proportion. Compare the ratio of a known pair of corresponding sides to the ratio of the unknown side and its corresponding side. Let \( x \) represent the length of side \( ST \).

\[ \frac{LN}{RT} = \frac{MN}{ST} \]

\[ \frac{8}{10} = \frac{7}{x} \]

Use cross products to solve for \( x \).

\[ 8x = 10 \cdot 7 \]

\[ 8x = 70 \]

\[ x = 8.75 \]

The length of side \( ST \) is 8.75 units.

Question 79 (page 182)

The correct answer is 21.

The rectangles are similar, so the lengths of the corresponding sides are proportional. Let \( w \) represent the missing width.
The larger poster board is 21 inches wide.

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The ratio of the area of the larger circle to that of the smaller circle is $9:4$.

**Question 80 (page 182)**

**D** Correct. If the dimensions of two similar figures are in the ratio $a:b$, then their areas will be in the ratio $(a/b)^2$. The diameters of the two circles are in the ratio of $3:2$. The area of the two circles will then be in the ratio $3^2:2^2 = 9:4$. The ratio of the area of the larger circle to that of the smaller circle is $9:4$.

**Question 81 (page 183)**

**A** Correct. When you multiply the dimensions of a figure by a scale factor, the perimeter of the figure changes by the scale factor. In this case, the scale factor is $2:5$.

To find the perimeter of triangle $PQR$, multiply the perimeter of triangle $MNO$ by the scale factor $2:5$.

$$45 \cdot \frac{2}{5} = 18 \text{ cm}$$

The perimeter of triangle $PQR$ is 18 centimeters.

**Question 82 (page 183)**

**D** Correct. When you multiply the dimensions of a solid figure by a scale factor, the volume of the figure changes by the cube of that scale factor. The scale factor is 3. The cube of the scale factor is $3^3 = 27$.

To find the volume of the larger tank, multiply the volume of the smaller tank by 27.

$$300 \cdot 27 = 8100 \text{ gal}$$

The volume of the larger tank is 8100 gallons.

**Question 83 (page 183)**

**B** Correct. If the dimensions of two similar solid figures are in the ratio $a:b$, then their volumes will be in the ratio $(a/b)^3$. The dimensions of the two figures are in the ratio $2:3$. The ratio of their volumes will be the cube of that ratio.

$$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

To find the volume of the smaller box, multiply the volume of the larger box by $\frac{8}{27}$.

$$162 \cdot \frac{8}{27} = 48 \text{ in}^3$$

The volume of the smaller box is 48 cubic inches.

**Question 84 (page 201)**

**B** Correct. Use a proportion to determine how many hours Rhonda actually spent on the project. Let $n$ represent the number of hours Rhonda actually spent on the project. Write a proportion.

$$\frac{12}{100} = \frac{80}{n}$$

$$80n = 12 \cdot 100$$

$$80n = 1200$$

$$n = 15$$

Rhonda actually spent 15 hours on the project.

**Question 85 (page 201)**

**B** Correct. Use a proportion to solve this rate problem. Write two ratios, each of which compares the number of switches to the number of hours. Let $x$ equal the number of hours required to manufacture 210 switches.
MATHEMATICS

Question 86 (page 201)
A Correct. The events are dependent because the outcome of the first draw affects the outcome of the second draw. Find the probability of the first event. For the first draw, 4 of the 25 possible outcomes are numbers less than five: 1, 2, 3, and 4. The probability that the first number drawn will be less than 5 is \( \frac{4}{25} \).

\[ P(1st < 5) = \frac{4}{25} \]

Find the probability of the second event. One number was drawn, so now there are only 24 numbers in the box. If the first number drawn was a number less than 5, only 3 of the 24 possible outcomes can be numbers less than 5. The probability that the second number drawn will be less than 5 is \( \frac{3}{24} \).

\[ P(2nd < 5) = \frac{3}{24} \]

The probability that both numbers will be less than 5 is equal to the product of the two probabilities.

\[ P(1st < 5 \text{ and } 2nd < 5) = P(1st < 5) \cdot P(2nd < 5) = \frac{4}{25} \cdot \frac{3}{24} = \frac{12}{600} = \frac{1}{50} \]

Question 87 (page 201)
C Correct.

In 811 times at bat, Reggie got 210 + 20 + 1 + 6 = 237 hits. The experimental probability that Reggie will get a hit during his next time at bat is the ratio of the number of times he got a hit (237) to the number of trials (811).

\[ \frac{237}{811} = 0.292 \]

The probability that Reggie will get a hit during his next time at bat is 0.292.

Question 88 (page 202)
C Correct.

Use the experimental probability to predict the number of seeds that will be 2 inches or more in height.

There were a total of \( \frac{9 + 16 + 26 + 24}{4} = 75 \) plants in this experiment. Of these, \( \frac{66}{75} \) or \( \frac{22}{25} \) plants reached a height of at least 2 inches after 3 months. The experimental probability that a plant will reach a height of at least 2 inches is \( \frac{66}{75} \), or \( \frac{22}{25} \).

Let \( n \) represent the number of plants out of 600 that will reach a height of at least 2 inches. Write a proportion.

\[ \frac{n}{600} = \frac{22}{25} \]

Therefore, 528 plants out of 600 can be expected to reach a height of at least 2 inches in 3 months.

Question 89 (page 202)
C Correct. The median is the middle value when the data elements are listed in order. Half the scores are above the median, and half the scores are below it. Therefore, Jean should use the median of the data.

Question 90 (page 203)
D Correct.

Trish spent a total of \( \frac{600 + 200 + 360 + 40 + 50 + 250}{6} = \$1500 \).

The plane fare should be \( \frac{600}{1500} = \frac{2}{5} = 0.4 \), or 40% of the graph. This is larger than \( \frac{3}{4} \), or 25%, of the graph and less than \( \frac{1}{2} \), or 50%, of the graph.

The cost of food should be \( \frac{200}{1500} = \frac{2}{15} \approx 0.13 \), or 13% of the graph. Together, plane fare and food should be 40% + 13% = 53%, or slightly more than half the graph. Hotel costs should be \( \frac{360}{1500} = \frac{6}{25} = 0.24 \), or 24% of the graph. Hotel costs will be slightly less than \( \frac{3}{4} \), or 25%, of the graph.

Only choice D fits these requirements.
Question 91 (page 204)

B Correct. The January bar is slightly above 1000, at approximately 1100. The February bar is less than halfway between 2000 and 3000, at approximately 2400. The March bar is slightly above 3000, at approximately 3100. The April bar is between 2000 and 3000, at approximately 2600; it is higher than the February bar.

Question 92 (page 205)

A Incorrect. The mean gives the average of a set of numbers, not the most frequently occurring data element.
B Incorrect. The median gives the middle value of a set of numbers, not the most frequently occurring data element.
C Correct. The mode tells which value occurs most frequently. In this case the mode is 7. The value 7 occurs 5 out of the 8 times listed. It shows that Ava most frequently spends 7 hours studying during a week.
D Incorrect. The range measures the spread between the highest and lowest values in the data, not the most frequently occurring data element.

Question 93 (page 205)

A Incorrect. Sales increased at a constant rate of 5 CDs per week for the first three weeks, but they then decreased during the fourth week.
B Correct. Calculate the mean number of CDs sold per week.
\[
\frac{280 + 285 + 290 + 285 + 295 + 300}{6} = 289.16 \approx 289
\]
C Incorrect. The length of the bar for Week 6 is three times the length of the bar for Week 1. However, read the numerical values of the bars from the graph. The value for the bar for Week 1 is 280. The value of the bar for Week 6 is 300. Since 300 ≠ 3 • 280, one bar is not three times as large as the other.
D Incorrect. Calculate the total number of CDs sold.
\[
280 + 285 + 290 + 285 + 295 + 300 = 1735
\]
Since 1735 > 1800, the store did not sell more than 1800 CDs during the six-week period.

Question 94 (page 220)

C Correct. Calculate the cost of the DVD player at Store A with the 10% discount.
Use a proportion to find 10% of 119.
\[
\frac{10}{100} = \frac{x}{119}
\]
\[
100x = 1190
\]
\[
x = 11.9
\]
10% of $119 is $11.90.
\[
119.00 - 11.90 = 107.10
\]
The cost of the DVD player at Store A with the 10% discount is $107.10.

Calculate the cost of the DVD player at Store B with the 15% off coupon.
Use a proportion to find 15% of 138.
\[
\frac{15}{100} = \frac{x}{138}
\]
\[
100x = 2070
\]
\[
x = 20.7
\]
15% of $138 is $20.70.
\[
138.00 - 20.70 = 117.30
\]
The cost of the DVD player at Store B with the 15% discount is $117.30.
This is more than the cost at Store A. Subtract to find the difference.
\[
117.30 - 107.10 = 10.20
\]
The DVD player costs $10.20 more at Store B than at Store A.

Question 95 (page 220)

D Correct. One way to solve this problem is to find the number of tiles needed to fill the large rectangle and then subtract the number of tiles that would have been needed for the small rectangle that will not be tiled.
If the tiles are 3 inches on a side, there are 4 tiles per foot (3 • 4 = 12).
The width of the large rectangle is 2.5 feet. Multiply 4 tiles per foot by 2.5 feet to find the number of tiles needed for the width.
\[
4 \times 2.5 = 10
\]
The length of the large rectangle is 6 feet. Multiply 4 by 6 to find the number of tiles needed for the length.

\[ 4 \times 6 = 24 \]

Multiply the number of tiles needed for the width by the number of tiles needed for the length to find the number of tiles needed to fill the large rectangle.

\[ 10 \times 24 = 240 \]

In the same way, find the number of tiles needed to fill the small rectangle.

Number of tiles needed for the width: 4 \cdot 1.5 = 6

Number of tiles needed for the length: 4 \cdot 2 = 8

Number of tiles needed to fill the small rectangle: 6 \cdot 8 = 48

Subtract the tiles needed to fill the small rectangle from the tiles needed to fill the large rectangle to find the total number of tiles needed.

\[ 240 - 48 = 192 \text{ tiles} \]

Jessica will need 192 tiles. The tiles are sold in boxes of 100 tiles per box; she will need 2 boxes of tiles.

Question 96 (page 220)

B Correct.

The cost of producing 250 staplers can be found by evaluating the original function for \( n = 250 \).

\[ c = 1.12n + 300 \]

\[ c = 1.12 \times 250 + 300 \]

\[ c = 280 + 300 \]

\[ c = 580 \]

If the fixed costs increase by 15%, use a proportion to find 15% of $300.

\[ \frac{15}{100} = \frac{x}{300} \]

\[ 100x = 4500 \]

\[ x = 45 \]

Find the percent by which the cost of producing 250 staplers increased.

\[ 625 - 580 = 45 \]

What percent is 45 of 580? Write a proportion.

\[ \frac{x}{100} = \frac{45}{580} \]

\[ 580x = 4500 \]

\[ x = 7.76 \]

The cost of producing 250 staplers increased by about 8%.

Question 97 (page 220)

C Correct. If Jesse and Philippe start at the same time and meet along the road, they both travel the same amount of time. Let \( t \) represent the time each travels. The rate they each travel multiplied by the time they each travel is equal to the distance they each travel (\( d = rt \)). The distance Jessie travels plus the distance Philippe travels equals 5 miles. Write an equation that shows the sum of the distances they travel equals 5 miles.

\[ 4t + 6t = 5 \]

\[ 10t = 5 \]

\[ t = 0.5 \]

They will meet in 0.5 hour, or 30 minutes.

Question 98 (page 221)

B Correct.

Find the minimum reduction factor for the height. Multiply by 12 to convert the height of the original painting to inches.

\[ 3 \text{ ft} \times 12 \text{ in/ft} = 36 \text{ in.} \]

The height of the original painting is 36 inches. Find the scale factor by which it would have to be reduced if it is to fit in a 9-inch-tall space.

\[ \frac{9}{36} = \frac{1}{4} \]

The height of the reproduction would need to be reduced by a factor of \( \frac{1}{4} \).

Find the minimum reduction factor for the width. Multiply by 12 to convert the width of the original painting to inches.

\[ 2 \text{ feet} \times 12 \text{ in/ft} = 24 \text{ in.} \]

The width of the original painting is 24 inches. Find the scale factor by which it would have to be reduced if it is to fit in an 8-inch-wide space.
The width of the reproduction would need to be reduced by a factor of $\frac{1}{3}$.

One dimension must be reduced by a factor of $\frac{1}{3}$ to fit in the allotted space, and the other dimension must be reduced by a factor of $\frac{1}{4}$. Since $\frac{1}{4}$ is the smaller reduction factor, the picture must be reduced by a factor of $\frac{1}{4}$ at minimum if both dimensions are to fit on the page.

**Question 99 (page 221)**

**C Correct.** The function $h = 12 - 0.1m$ when rewritten as $h = -0.1m + 12$ is in the form $y = mx + b$, so it is a linear function.

To find the height of the candle when it started burning at $m = 0$, substitute 0 for $m$ in the function.

\[ h = 12 - (0.1)(0) \]
\[ h = 12 - 0 \]
\[ h = 12 \]

The candle was 12 inches tall when it started burning.

To find the height of the candle when $m = 120$ minutes, substitute 120 for $m$ in the function.

\[ h = 12 - (0.1)(120) \]
\[ h = 12 - 12 \]
\[ h = 0 \]

The candle can burn for at most 120 minutes because in that amount of time its height will have been reduced to 0 inches.

If the height of the candle were directly proportional to the number of minutes it burned, the function comparing them would be in the form $y = mx$. The function $h = 12 - 0.1m$ is not in this form.

**Question 100 (page 221)**

**A Correct.** Let $x$ represent the cost of the shirt. Then $2x$ represents the cost of the pants. The sum of the costs of the three items, shirt + pants + shoes, equals the total cost, $100. Use the equation $x + 2x + 40 = 100$. If like terms are simplified, this is the same equation as $3x + 40 = 100$.

**B Incorrect.** Let $x$ represent the width of the rectangle. The length is 40 more than three times the width. The length is $3x + 40$.

To find the perimeter, use the formula $P = 2(l + w)$.

\[ 100 = 2(3x + 40 + x) \]
\[ 100 = 2(4x + 40) \]
\[ 100 = 8x + 80 \]

Use the equation $100 = 8x + 80$ to solve this problem.

**C Incorrect.** Let $x$ represent the number of years. To solve this problem, use the simple interest formula $I = prt$. The interest rate is 3%, or 0.03.

Use the equation $40 = 100(0.03)(x)$, or $40 = 3x$, to solve this problem.

**D Incorrect.** Let $x$ represent the number of cups sold. If the student council earns $3 for each cup sold, $3x$ represents the amount they earn for selling $x$ cups. To find the profit, subtract the amount they spent on the cups. Use the equation $3x - 40 = 100$ to solve this problem.

**Question 101 (page 221)**

**C Correct.** One way to find the values of $x$ for which $f(x) \geq g(x)$ is to write the following inequality and simplify it.

\[ f(x) \geq g(x) \]
\[ 2(x - 3) + 7 \geq \frac{1}{2}x + 7 \]
\[ 2x - 6 + 7 \geq \frac{1}{2}x + 7 \]
\[ 2x + 1 \geq \frac{1}{2}x + 7 \]

**Question 102 (page 222)**

**B Correct.** The graphs show a pattern. All the graphs are parabolas, and all are congruent. The $x$-intercepts of the graphs equal the value by which $x$ is decreased in the function.

The function $y = (x - 1)^2$ intersects the $x$-axis at the point $x = 1$.

The function $y = (x - 3)^2$ intersects the $x$-axis at the point $x = 3$.

The function $y = (x - 5)^2$ intersects the $x$-axis at the point $x = 5$.

Based on this pattern, the function $y = (x - 2)^2$ should intersect the $x$-axis at the point $x = 2$.

**Question 103 (page 223)**

**A Correct.**

Look for the pattern. The dimensions of the cylinders are increasing by a scale factor of 1.5.

\[ \frac{3}{2} = \frac{4.5}{3} = \frac{6.75}{4.5} = 1.5 \]
To find the volume of the previous cylinder in this series, you could increase its dimensions by the same scale factor, 1.5.

Question 104 (page 223)
B  Correct. Represent the number of visitors the site had each week in terms of $x$.
   First Week: $x$
   Second Week: $2x$
      (twice as many as the first week)
   Third Week: $2x + 22$
      (22 more than the second week)
   Fourth Week: $2.5x$
      (2.5 times as many as the first week)

The mean of the number of visitors is equal to the sum of the number of visitors divided by the number of weeks for which records were kept, 4.

Represent their sum.
   \[ \text{1st wk} + \text{2nd wk} + \text{3rd wk} + \text{4th wk} = \]
   \[ x + 2x + (2x + 22) + 2.5x \]

Represent their mean.
   \[ \frac{x + 2x + (2x + 22) + 2.5x}{4} \]
How is the science section organized?

Five objectives are tested on the Grade 10 science TAKS test. The science section of this study guide is therefore organized into five main parts, one for each objective.

- Objective 1: The Nature of Science
- Objective 2: The Organization of Living Systems
- Objective 3: The Interdependence of Organisms and the Environment
- Objective 4: The Structures and Properties of Matter
- Objective 5: Motion, Forces, and Energy

For each objective there is a review and a set of practice questions. Start by reading the review for each objective. After you read the review, you can test your knowledge of the objective by trying the practice questions.

Will this study guide tell me everything I need to know about science?

No, but it’s a great place to review what you’ve learned in school. This study guide explains some, but not all, of the science ideas that you need to know and understand. You can also increase your science knowledge by studying:

- Science books from your school or library
- Notes from your science classes
- Science tests, quizzes, and activity sheets
- Laboratory reports and notes from field investigations

What kinds of practice questions are in the Science Study Guide?

The Science Study Guide contains questions similar to those found on the Grade 10 science TAKS test. There are three types of questions in the science section.

- **Multiple-Choice Questions**: Most of the practice questions are multiple-choice items with four answer choices. Many of these questions follow a short passage, a chart, a diagram, or a combination of these. Read each passage carefully. If there is a chart or diagram, study it. Passages, charts, and diagrams usually contain details and other information that will help you answer the question. Then read the question carefully and consider what you are being asked. Read each answer choice before you choose the best answer.

  It’s always a good idea to reread the question after you have thought about each answer choice.

- **Griddable Questions**: Some questions use an eight-column answer grid like the ones used on the Grade 10 mathematics TAKS test. Griddable questions ask you to measure something or use math to solve a science problem. You will see an example of a griddable question on page 303.

- **Cluster Questions**: Some multiple-choice questions are grouped together in clusters. Each cluster begins with a stimulus that may include a passage, a diagram, a chart, or a combination of these. The information in the stimulus will help you focus on the cluster questions.
The stimulus is followed by two to five multiple-choice questions. The cluster questions usually test several different science objectives, but they are all related to the stimulus. To answer the cluster questions, you will need to use information from the stimulus, as well as your own knowledge of science, so read and study the stimulus carefully before you answer the cluster questions. Then think about what you already know from your study of science. You will see examples of science clusters on pages 324–327.

**How do I use an answer grid?**

The answer grid contains four columns of numerals followed by a fixed decimal point and three additional columns of numerals. Your answer will always be limited to a number from 0 to 9,999.999.

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This is the grid found on the actual test.

Let’s say you are asked to calculate the force needed to accelerate an object to a velocity of 1.8 meters per second. If your answer is 288.6 newtons, you should write the digit 2 at the top of the hundreds column, an 8 in the tens column, an 8 in the ones column, and a 6 in the tenths column. Be careful to record the digits in the column with the correct place value with respect to the decimal point. Then fill in the bubbles that correspond to your answer. Find the correct bubbles and darken the circles. Check to make sure that you bubbled in the same number that you wrote at the top of each column.

Extra zeros before or after the answer will not affect your score.

**How will I know whether I answered the practice questions correctly?**

The answers to the practice questions are in an answer key at the back of the science section (page 333). For most questions, the answer key explains why each answer choice is correct or incorrect. After you answer the practice questions, you can check your answers to see how you did. If you chose the wrong answer to a question, carefully read the answer explanation to find out why your answer is incorrect. Then read the explanation for the correct answer.

If you still do not understand the correct answer, ask a friend, family member, or teacher for help. When you choose the correct answer, it is still a good idea to read the answer explanation. It may help you better understand why the answer is correct.
Is there anything else in the science section?

Yes! A formula chart is provided on page 248 of this study guide. It is identical to the formula chart that is provided to you when you take the Grade 10 science TAKS test. You will need the formula chart to answer some of the practice questions. The good news is that you don't have to memorize the formulas, constants, and conversions. However, you do need to know how to use them to solve science problems. Remember, knowing which formula to use is just as important as knowing how to use it. You'll learn more about formulas, constants, and conversions in the review for Objectives 4 and 5. The formula chart also contains a 20 centimeter ruler.

A periodic table of the elements is provided on page 249. An identical periodic table is provided when you take the Grade 10 science TAKS test. You will need information from the periodic table to answer some of the practice questions. You'll learn more about the periodic table in the review for Objective 4.

In addition to the materials on pages 248 and 249, a tear-out copy of the formula chart and the periodic table is provided at the back of this study guide.

There is a science activity called “The Floating Rubber Band: A Scientific Trick” on page 328. You can do this activity at home. It will help you practice and strengthen some of the science skills that you'll review in Objective 4 (The Structures and Properties of Matter). The review for Objective 4 begins on page 290. After you complete the activity, you can compare your results with the sample results on page 343.

Many of the review pages contain clipboards. The clipboards contain tips, helpful information, important facts, and interesting details.

Remember! clipboards contain information that you have probably learned before. They are reminders to help refresh your memory.

Did you know? clipboards contain fun science facts that are probably not familiar to you.

How do I use this study guide?

Carefully read the review section. If you do not understand something, ask for help. Then answer the practice questions. Use the answer key at the back of the science section to check your answers. It is a good idea to read all five reviews and answer all the practice questions even if you passed some of these objectives. Study at a pace that is comfortable for you. The Science Study Guide contains a lot of information. If you plan to read all the reviews and answer all the practice questions, you may want to allow yourself several weeks.
### FORMULA CHART
for Grades 10–11 Science Assessment

<table>
<thead>
<tr>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density = ( \frac{\text{mass}}{\text{volume}} )</td>
</tr>
<tr>
<td>Heat gained or lost by water = ( (\text{mass in grams})(\text{change in temperature})(\text{specific heat}) )</td>
</tr>
<tr>
<td>Speed = ( \frac{\text{distance}}{\text{time}} )</td>
</tr>
<tr>
<td>Acceleration = ( \frac{\text{final velocity} - \text{initial velocity}}{\text{change in time}} )</td>
</tr>
<tr>
<td>Momentum = ( \text{mass} \times \text{velocity} )</td>
</tr>
<tr>
<td>Force = ( \text{mass} \times \text{acceleration} )</td>
</tr>
<tr>
<td>Work = ( \text{force} \times \text{distance} )</td>
</tr>
<tr>
<td>Power = ( \frac{\text{work}}{\text{time}} )</td>
</tr>
<tr>
<td>% efficiency = ( \frac{\text{work output}}{\text{work input}} \times 100 )</td>
</tr>
<tr>
<td>Kinetic energy = ( \frac{1}{2} (\text{mass} \times \text{velocity}^2) )</td>
</tr>
<tr>
<td>Gravitational potential energy = ( \text{mass} \times \text{acceleration due to gravity} \times \text{height} )</td>
</tr>
<tr>
<td>Energy = ( \text{mass} \times (\text{speed of light})^2 )</td>
</tr>
<tr>
<td>Velocity of a wave = ( \text{frequency} \times \text{wavelength} )</td>
</tr>
<tr>
<td>Current = ( \frac{\text{voltage}}{\text{resistance}} )</td>
</tr>
<tr>
<td>Electrical power = ( \text{voltage} \times \text{current} )</td>
</tr>
<tr>
<td>Electrical energy = ( \text{power} \times \text{time} )</td>
</tr>
</tbody>
</table>

### Constants/Conversions

- \( g = \text{acceleration due to gravity} = 9.8 \text{ m/s}^2 \)
- \( c = \text{speed of light} = 3 \times 10^8 \text{ m/s} \)
- Speed of sound = 343 m/s at 20°C
- 1 cm\(^3\) = 1 mL
- 1 wave/second = 1 hertz (Hz)
- 1 calorie (cal) = 4.18 joules
- 1000 calories (cal) = 1 Calorie (Cal) = 1 kilocalorie (kcal)
- newton (N) = kgm/s\(^2\)
- joule (J) = Nm
- watt (W) = J/s = Nm/s
- volt (V) = ampere (A) = ohm (Ω)
### Periodic Table of the Elements

**Group**

<table>
<thead>
<tr>
<th>Period</th>
<th>Group</th>
<th>Symbol</th>
<th>Atomic Number</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>H</td>
<td>1</td>
<td>Hydrogen</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>He</td>
<td>2</td>
<td>Helium</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>Li</td>
<td>3</td>
<td>Lithium</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>Be</td>
<td>4</td>
<td>Beryllium</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>B</td>
<td>5</td>
<td>Boron</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>C</td>
<td>6</td>
<td>Carbon</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>N</td>
<td>7</td>
<td>Nitrogen</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>O</td>
<td>8</td>
<td>Oxygen</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>F</td>
<td>9</td>
<td>Fluorine</td>
</tr>
</tbody>
</table>

**Mass numbers in parentheses are those of the most stable or most common isotope.**

**Lanthanide Series**

<table>
<thead>
<tr>
<th>Period</th>
<th>Group</th>
<th>Symbol</th>
<th>Atomic Number</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>13</td>
<td>Al</td>
<td>13</td>
<td>Aluminum</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>Si</td>
<td>14</td>
<td>Silicon</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>P</td>
<td>15</td>
<td>Phosphorus</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
<td>S</td>
<td>16</td>
<td>Sulfur</td>
</tr>
</tbody>
</table>

**Actinide Series**

<table>
<thead>
<tr>
<th>Period</th>
<th>Group</th>
<th>Symbol</th>
<th>Atomic Number</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>17</td>
<td>Cl</td>
<td>17</td>
<td>Chlorine</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>Ar</td>
<td>18</td>
<td>Argon</td>
</tr>
</tbody>
</table>

**Symbol**

- H: Hydrogen
- He: Helium
- Li: Lithium
- Be: Beryllium
- B: Boron
- C: Carbon
- N: Nitrogen
- O: Oxygen
- F: Fluorine
- Ne: Neon

**Atomic number**

- 1: Hydrogen
- 2: Helium
- 3: Lithium
- 4: Beryllium
- 5: Boron
- 6: Carbon
- 7: Nitrogen
- 8: Oxygen
- 9: Fluorine

**Mass numbers in parentheses are those of the most stable or most common isotope.**

**Mass numbers**

- 26.086: Silicon
- 30.974: Sulfur
- 32.066: Sulfur

**Revision**

- Revised October 15, 2001
Objective 1
The student will demonstrate an understanding of the nature of science.

From your studies in science, you should be able to demonstrate an understanding of the nature of science.

Nature of science? What’s that?
Science is one way that people make sense of the world. Science involves asking questions about the natural world and finding ways to answer them. That’s what we mean by the nature of science.

Here are just a few of the many ways to answer scientific questions: by observing the natural world, by performing experiments, by completing investigations, by doing library research, or by building models. As you can see, science is more than just a subject you study in school.

How do I do a science investigation?
First, observe your surroundings and ask questions about what you see. A good question for a science experiment is one that you can answer based on facts. Let’s come up with one.

I know that plants need light to grow. Hmm . . . how about this for our question: Do plants grow better in some colors of light than in others? We can perform an experiment by growing plants in different colors of light. We can gather facts from our experiment to help us answer our question.

Here’s a question that we couldn’t use for a science investigation: Which type of music sounds better, rap or techno? The answer to the question depends on the opinion of the person answering it. Some people would say rap; others would say techno. We could gather only opinions about the answer to this question, not facts.
O.K., we have a question. How do we set up an experiment to answer it?

We start by making a plan. First let’s narrow things down a bit. We can’t really test every possible color of light, so let’s pick three: red, blue, and green. And we can’t test every type of plant that there is. Let’s stick to just one. How about tomato plants?

Now let’s think about the materials we’re going to need. We’ll need tomato plants, potting soil, and foam cups that we can use as pots. We’ll need a way to produce different colors of light. There are several ways we could do this. We could use colored lightbulbs or different colors of plastic wrap.

We’ll also need a way to make sure that our plants receive only the colors of light that we want them to. The easiest way would be to find some boxes that are big enough to set over the plants. Then we could cut off the tops of the boxes and cover them with plastic wrap in different colors.

We have our materials. Are we ready to start our experiment?

Not quite yet. First we need to decide how we are going to control our variables. A variable is anything in an experiment that we can change. We want to keep all the variables the same except the one we are trying to test. In our experiment the variable that we are testing, the one that we will change, is the color of light. We want to keep everything else about our plants the same so that we can be fairly certain that any differences in the plants’ growth are caused only by the color of light the plants receive.

To control our variables, we’re going to place two tomato plants in containers of the same size and in the same type of soil. Every other day we’ll give each of the containers the same amount of water.

We’ll put one container in a box covered with red plastic wrap, one in a box covered with blue plastic wrap, and one in a box covered with green plastic wrap. These will be the three experimental plants. We’ll put all the boxes together on a sunny porch so that they’ll all receive the same number of hours of light.

Remember!
White light is a mixture of all colors of light. Sunlight can be separated into the colors of a rainbow because sunlight is white light.
Oh, I almost forgot. We’re also going to need a control.

**What’s a control?**

A *control* is something that we can compare our experimental results to. Because we’re changing the color of light that the plants receive, our control can be a plant that receives all colors of light. Let’s set up a fourth box and cover it with clear plastic wrap. The plant in this box will receive all colors of light and will act as our control.

**I think you forgot something else. Don’t we need to have a hypothesis?**

Yes, we do! Do you remember the question we’re trying to answer? *Do plants grow better in some colors of light than in others?* Let’s restate our question as a *hypothesis*. A hypothesis is an educated guess about what we think will happen in our experiment based on our scientific knowledge or research. A hypothesis is also a statement that we can test. I’m going to base our hypothesis on something I already know about plants. I know plants are green, so I’m going to hypothesize that *plants will grow best in green light*. This is a statement that we will be able to test with our experiment.

**What data do we need to collect, and how are we going to collect it?**

A bunch of numbers and notes written on scraps of paper is hard to work with. So we need to get organized. First let’s determine what kind of data we’re going to collect. We need a way to measure plant growth. We could measure the heights of the plants or count their leaves or even measure the thickness of the stems. In science there’s almost always more than one way to solve a problem or find the answer to a question.

Let’s use the heights of our plants as a measure of their growth. We’ll measure the heights of the plants once a week for four weeks (Weeks 1–4). We’ll also measure the heights of the plants at the beginning of the experiment (Week 0). To keep up with our plant height data, we’ll make a chart. We’ll have one row for each week and one column for each light color. Remember, this is not the only way to organize our data.

I think we’re ready now. Let the experimenting begin!
Finally! If I measure the plants, will you record the measurements in our chart?

Sure. After four weeks this is what our data chart looks like.

<table>
<thead>
<tr>
<th>Average Plant Height</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Red Light</td>
</tr>
<tr>
<td>Week 0</td>
</tr>
<tr>
<td>Week 1</td>
</tr>
<tr>
<td>Week 2</td>
</tr>
<tr>
<td>Week 3</td>
</tr>
<tr>
<td>Week 4</td>
</tr>
</tbody>
</table>

So now we have lots of data. What do we do next?

We want to be able to use the data in the chart to draw a conclusion about what happened in our experiment. But it can sometimes be difficult to do this when the data are organized in a chart. Let's take our information and graph it. A graph really helps show trends. We can use a graph to compare the heights of our plants more easily.

Two types of graphs that are often used in science are line graphs and bar graphs. Line graphs are often used to show how one variable in an experiment changes over time. Bar graphs are used to display data in separate categories.

For our experiment we could use a line graph to show how the heights of the plants changed over time, or we could use a bar graph to show the heights of our four plants at the end of the experiment. Both would be good ways to show the data in our chart.

How do I set up a graph?

There are a few rules about what goes where on a graph. In general, the independent (or manipulated) variable is plotted on the horizontal axis (x), and the dependent (or responding) variable is plotted on the vertical axis (y). The independent variable is the variable that you change during the experiment. In our experiment the independent variable was the color of light the plants received.

The dependent variable is the one that changes as a result of the independent variable. The dependent variable is the one that you measure or observe during the experiment. In our experiment the dependent variable was the heights of the plants.
How would the data look if we used a bar graph?

Here is a bar graph of some of the data from our experiment. This graph shows the average heights of the plants after four weeks.

![Bar graph showing effect of light color on tomato-plant height]

**Effect of Light Color on Tomato-Plant Height**
**After Four Weeks of Growth**

- **Red light**
- **Blue light**
- **Green light**
- **White light (control)**

How would our data look if we used a line graph?

Let’s use a line graph to show how the plants’ heights changed during the last two weeks of the experiment. In this case time is the independent variable, and average plant height is the dependent variable. We’ll use a different line for each light color.

![Line graph showing effect of light color on heights of tomato plants]

**Effect of Light Color on Heights of Tomato Plants**

- **Red light**
- **Blue light**
- **Green light**
- **White light (control)**

Average Plant Height (cm)

Time (week)
So we’ve made graphs from our data. What are we supposed to do with the graphs?

Now that the data are in a graph form, we can analyze the data and make conclusions. If you look at the graphs, you should be able to see some patterns. For instance, we can see that the plants grew to different heights, so we can conclude that the color of light that the plants received may have had an effect on their growth. In order to be certain we would have to repeat this experiment many times.

We can also see that the tallest plants grew in blue light, the second-tallest in red light, the third-tallest in white light, and the shortest in green light. The tomato plants grown in red and blue light grew more than our control, and the plants grown in green light grew less than our control.

Wow! And we thought the green light would make the plants grow best. I guess our experiment was a waste of time, huh?

Of course it wasn’t a waste of time! It’s true that our data don’t support our hypothesis, but that’s O.K. A hypothesis is an educated guess; it doesn’t have to be correct. Though we weren’t able to support our hypothesis, we discovered that the tomato plants that grew in the blue and red light were taller. This is important information that can lead to a new hypothesis to test.

If the plants that received green light had grown taller than the others, could we have said that our hypothesis is true?

No. We performed only a single experiment, and we used only eight plants. If we repeated our experiment, we might get slightly different results. In addition, we tested only tomato plants. If we tested other types of plants, we might find that different types of plants grow best in different colors of light.

Is a hypothesis kind of like a theory?

A theory is a general explanation of a set of observations about the natural world. A theory helps explain how things happen the way they do in nature. Unlike a hypothesis, a theory is supported by lots of data collected from many different experiments and observations.

Theories can change over time as scientists gather more evidence. Hundreds of years ago scientists proposed a theory called spontaneous generation. This theory stated that some living things could develop from nonliving materials. For instance, it was thought that maggots could form from rotting meat. However, as scientists made more observations about the world around them, the theory of spontaneous generation was eventually replaced with the theory of biogenesis. The theory of biogenesis states that living things can come only from other living things. For example, it is now known that maggots hatch from tiny eggs that flies lay on rotting meat.
Can our data tell me why the tomato plants grew taller in red and blue light than in green light?

Unfortunately, our data do not answer that question. We would need to do much more research to find out how light affects plant growth. We would need to learn about the properties of red, blue, and green light. We would also need to understand how light is reflected and absorbed by plants and how plants use the light that they absorb. Maybe you could come up with another investigation!

I want to hear about an experiment in which the scientist made mistakes. Can you tell me about one?

Sure! Scientists make as many mistakes as other people do. Let me tell you about an experiment that I did. I’ll describe it and let you try to figure out what was wrong with it.

I wanted to figure out whether music affects plant growth. I hypothesized that classical music would make plants grow taller than jazz or pop music. I started with tomato plants that were the same age and of the same type. I gave the plants the same amount of water every other day and measured their heights each day for one month.

I put one plant in my living room window, one in my kitchen window, one in my bedroom window, and one in the school greenhouse. Each day I exposed the living room plant to six hours of classical music, the kitchen plant to six hours of jazz music, and the bedroom plant to six hours of pop music. The plant in the greenhouse wasn't exposed to any music at all.

Did You Know?

Astronomer Leonard Amburgey typed the wrong numbers into the computer controlling his telescope, causing him to accidentally discover an asteroid that passes close to Earth.
So what happened? What kind of results did you get?

Well, I found that the plant with no music grew tallest, followed by the plant with classical music. The third-tallest plant was the one with pop music, and the shortest plant was the one with jazz music.

Can you figure out the main mistake I made in designing this experiment?

Yes! You didn’t control some of your variables! The shapes and sizes of the windows are all different, and the windows are in different places.

You’re right! Because the plants are in different places, they probably received different amounts of light, so I had no way of knowing which was affecting the growth of the plants—the different amounts of light they received or the music they were exposed to.
Does that mean your experiment was a total failure? Did you learn anything at all from your data?

Well, I did learn that something caused the differences in the plants’ heights. I just can’t be certain what that something was. One thing I could do is repeat the experiment without playing music to any of the plants. If the results of this experiment were very similar to the results of my first experiment, I could conclude that the music played in the first experiment probably had very little effect on the plants’ growth. Instead, I could conclude that another factor, such as the amount of light, caused the differences in the plants’ heights.

Now I know how to set up an experiment, and I know science is important, but I’m not a scientist. Why do I need to know about science?

You might not end up in a career in which you wear a white lab coat and peer through a microscope all day. But no matter what you do, you’re still going to need science.

As you study science, you learn how to use and analyze the information that you are exposed to every day. Here are some examples of what you can do with a general knowledge of science: You can look at the nutrition labels on foods and make informed choices about your health. You can understand how your stereo sound system works. You can recognize which health and fitness programs sound too good to be true. You can solve problems in a logical way by looking at the facts and drawing conclusions.

With an understanding of science, you can also understand and make educated choices on complex issues—such as global warming, cloning, and modern medicine—that will affect the future of the human race.
**Objective 1**

**Now It’s Your Turn**

After you answer the practice questions, you can check your answers to see how you did. If you chose the wrong answer to a question, carefully read the answer explanation to find out why your answer is incorrect. Then read the explanation for the correct answer.

---

**Question 1**

A student observes a garden plot. Several plants shaded by a tree are not as large as those of the same species growing in the full sun. The student asks, “Why are the plants growing in the shade smaller than the plants growing in the sun?”

Which is the most reasonable hypothesis to explain the student’s observations?

A. The tree removes water from the soil. The drier soil limits the growth of those plants growing near the tree.

B. The tree removes minerals from the soil. The mineral-reduced soil limits the growth of those plants growing near the tree.

C. The tree blocks sunlight, lowering the light level in the shaded area. This limits the growth of this plant species, which grows best in direct sunlight.

D. The tree blocks sunlight, lowering the air temperature of the shaded area. This limits the growth of this plant species, which grows best at warmer temperatures.

---

**Question 2**

Paramecium caudatum was cultured first. Paramecium aurelia was added to the same culture one day later. According to the graph, which is the first day that *P. aurelia* will exist alone in the culture medium if the trend shown continues?

A. Day 12

B. Day 13

C. Day 16

D. Day 18

---

Answer Key: page 333
Arthropods such as blue crabs grow in spurts. Their growth is marked by a series of molts during which the old exoskeleton is shed and a new, larger one develops. A blue crab grows rapidly only during the period between shedding and developing a new exoskeleton. Which graph most likely represents the growth of a blue crab?

A student performed an experiment to study the effects of the amount of light and temperature on the rate of photosynthesis in elodea plants. The graph shows the data that were obtained. Which statement is not supported by the graph?

A. Low temperature reduces the rate of photosynthesis in elodea.
B. The rate of photosynthesis in elodea is affected by light intensity.
C. The rate of photosynthesis in elodea increases continually with increasing light intensity.
D. The highest rate of photosynthesis in elodea occurs at a temperature greater than 12°C.
**Question 5**

Which question cannot be answered by experimentation using a scientific approach?

A. Does hot water freeze faster than cold water?
B. Will wearing a copper bracelet reduce arthritis symptoms?
C. Do roses smell better than carnations?
D. Will popcorn kernels soaked in water produce a greater volume of popped corn than dry kernels?

**Question 6**

According to the nutrition label above, what is the total daily requirement for carbohydrates based on a 2000-Calorie diet?

A. 29 grams
B. 290 grams
C. 20 milligrams
D. 350 milligrams
Question 7

Research has shown an association between the use of aspirin or similar products and the development of a potentially deadly disease of the brain and liver called Reye's syndrome. This disorder sometimes occurs when aspirin or medicines containing salicylate compounds are administered during recovery from a viral infection, such as a cold, the flu, and chicken pox. Based on the information below, which nonprescription medicine is considered unsafe to take while recovering from a viral infection?

A  Multisymptom cold medicine

**ACTIVE INGREDIENTS** (1 fluid ounce)
- Doxylamine Succinate (12.5 mg),
- Dextromethorphan Hydrobromide (30 mg),
- Acetaminophen (1000 mg),
- Pseudoephedrine Hydrochloride (60 mg)

**INACTIVE INGREDIENTS**
- Alcohol, Citric Acid, FD&C Blue No. 1,
- FD&C Red No. 40, Flavoring, High-Fructose Corn Syrup, Polyethylene Glycol, Propylene Glycol, Purified Water, Saccharin Sodium, Sodium Citrate

B  Motion-sickness tablet

**ACTIVE INGREDIENT** (per tablet)
- Dimenhydrinate (50 mg)

**INACTIVE INGREDIENTS**
- Colloidal Silicon Dioxide, Croscarmellose Sodium, Lactose, Magnesium Stearate, Microcrystalline Cellulose

C  Medicine for an upset stomach

**ACTIVE INGREDIENT** (per tablespoon)
- Bismuth Subsalicylate (262 mg)

**INACTIVE INGREDIENTS**
- Benzoic Acid, Flavoring, Magnesium Aluminum Silicate, Methylcellulose, Red 22, Red 28, Saccharin Sodium, Salicylic Acid, Sodium Salicylate, Sorbic Acid, Water

D  Chewable antacid tablet

**ACTIVE INGREDIENTS** (per tablet)
- Famotidine (10 mg), Calcium Carbonate (800 mg), Magnesium Hydroxide (165 mg)

**INACTIVE INGREDIENTS**
- Cellulose Acetate, Corn Starch, Dextrates, Flavoring, Hydroxypropyl Cellulose, Hydroxypropyl Methylcellulose, Lactose, Magnesium Stearate, Pregalatinized Starch, Red Iron Oxide, Sodium Lauryl Sulfate, Sucrose

Answer Key: page 334
Objective 2

The student will demonstrate an understanding of the organization of living systems.

From your studies in biology, you should be able to demonstrate an understanding of the organization of living systems.

Living systems are organized?
All living things are made up of one or more cells that contain genetic material called DNA. In many organisms, cells are organized into tissues, organs, and organ systems. Organisms are organized into populations and communities. You need to be able to show that you understand the different levels of organization.

I know that organisms are made up of cells. But what do cells do? And how do they work?
Cells are the basic units of all living things. They carry out the life functions of an organism. In some ways, cells are a lot like factories. A furniture factory, for example, takes in raw materials such as wood, turns them into finished products such as chairs and tables, and then sends them out. The factory also needs a source of energy to run its tools, and it must get rid of wastes such as sawdust.

Cells take in raw materials, such as amino acids, change them into more-complex molecules such as proteins, and then transport these molecules to where they are needed. Cells produce energy for life processes by breaking down molecules like glucose. They also get rid of waste molecules produced by these processes.

So how do cells make things?
Cells don't make just anything. They make molecules. And the process by which cells make molecules is called synthesis. One important molecule synthesized by plant cells is glucose. Plant and animal cells use glucose as an energy source.
The synthesis of glucose by plant cells can be summed up with this chemical reaction:

\[
\begin{align*}
6\text{CO}_2 + 6\text{H}_2\text{O} \xrightarrow{\text{light}} & \text{C}_6\text{H}_{12}\text{O}_6 + 6\text{O}_2 \\
\text{carbon dioxide} + \text{water} \xrightarrow{\text{light}} & \text{glucose} + \text{oxygen}
\end{align*}
\]

In plants, this process takes place in organelles called chloroplasts. It requires light energy. For this reason, the process is called photosynthesis. (Photo- means “light.”)

Animal cells also use glucose as an energy source. However, animal cells can’t make glucose because they lack chloroplasts. So where do animals get glucose? They get it from eating plants or from eating animals that eat plants.

What about proteins? How are they synthesized?

Proteins are complex molecules that have many functions in living things. For instance, they are one of the main building materials in cells. Remember that proteins are made up of smaller units called amino acids. The process by which amino acids are linked together to form a protein is called protein synthesis.

Protein synthesis takes place on a cell’s ribosomes. All types of cells have ribosomes, even bacterial cells. This means that all types of cells are able to make their own proteins. Remind me to tell you more about protein synthesis once we’ve talked about DNA.
DNA? That’s genetic material, right?

Right! DNA carries the genetic information that controls the activities of a cell. It carries instructions that determine which proteins the cell will make. You inherited your DNA from your parents. Half of your DNA came from your mother, and half came from your father. A basic understanding of genetics will help you understand issues about your health, cloning, and other types of genetic engineering.

How does DNA “carry” genetic information?

Genetic information is carried within the DNA molecule itself—in its structure. DNA is made up of units called nucleotides. Each nucleotide has three parts: a sugar called deoxyribose, a phosphate group, and a nitrogen base. There are four different nitrogen bases in DNA: guanine (G), cytosine (C), adenine (A), and thymine (T).

A DNA molecule looks like a twisted ladder. This shape is often called a double helix. The diagram below models a small section of DNA that has been “flattened” so that you can see its parts.

The DNA “ladder” is made up of two strands of nucleotides. Sugars and phosphates make up the sides of the ladders. Each rung of the ladder is made up of two nitrogen bases, one from each strand. The same bases always bond with each other: cytosine with guanine, and adenine with thymine.
O.K., now I know about the parts of DNA, but I still don't see where the genetic information is. Can you show me?

Yes! In fact, you've already seen it. Look back at the model of DNA. The genetic information is coded in the order of the bases in a DNA molecule. For instance, if you look down the left side of the DNA structure, the order is ATGGCA.

I see it now! But how is this genetic code actually used for anything?

The genetic code is used as a blueprint to make proteins. First the genetic code in a section of DNA is transcribed to a molecule of mRNA. (Transcribed is a fancy way of saying “copied.”) RNA is very similar to DNA except that it:

- has only a single strand of nucleotides instead of two strands
- contains a different sugar (ribose instead of deoxyribose)
- contains the nitrogen base uracil (U) instead of thymine (T)

The “m” in “mRNA” stands for “messenger,” because mRNA copies genetic information from DNA (which is found in the nucleus) and carries it to another part of the cell (the ribosomes).

O.K., we've gone from DNA to mRNA. How do proteins fit into all this?

I'm getting to that. Think of the genetic code the mRNA is carrying as a series of three-letter “words.” Each of these three-letter words is called a codon. Different codons code for different amino acids. For example, the codon for the amino acid methionine is AUG (adenine, uracil, guanine). Another type of RNA, called tRNA (“t” stands for “transfer”) matches the codons in mRNA to the correct amino acids. As the mRNA strand moves along the ribosome, the amino acids are joined in the correct sequence to form a protein. This process is called translation.

**Translation of mRNA Codons**

<table>
<thead>
<tr>
<th>mRNA sequence</th>
<th>tRNA sequence</th>
<th>Amino acid sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUG</td>
<td>GUU</td>
<td>CCA</td>
</tr>
<tr>
<td>G</td>
<td>C</td>
<td>G</td>
</tr>
<tr>
<td>G</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>G</td>
</tr>
</tbody>
</table>

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Does the DNA sequence ever get messed up?

Good question. Yes, it does. When this happens, we call it a *mutation*. A mutation is a change in the nucleotide sequence of DNA. And as you can probably guess, a change in DNA leads to a change in mRNA, which can lead to a change in protein synthesis. Some mutations can be harmful or even fatal to an organism. However, most mutations have little or no effect on an organism.

Other types of mutations affect more than one codon. For instance, a mutation can cause one or more nucleotides to be added to or deleted from DNA. This type of mutation can lead to the production of a completely different protein. As a result, the mutation could be harmful, or even fatal, to the organism.

Aren’t there any good mutations?

Yes, there are! On rare occasions, a mutation can make an organism more likely to survive and reproduce. For example, a species of plant might produce a chemical with a scent that attracts pollinating flies. A mutation in one of the plants could make it produce a slightly different scent—one that is even more attractive to pollinators. This type of mutation would be beneficial to the plant.

However, even if a mutation benefits an organism, it may not be passed on to the organism’s offspring. For organisms that reproduce sexually, mutations can be passed to the next generation only if they occur in the organism’s sex cells (eggs or sperm).
You said that most mutations don’t have an effect on an organism. How can that happen?

For one thing, not all mutations lead to a different protein being made. There are more codons than there are amino acids. So, more than one codon can code for the same amino acid. The diagram below shows a codon chart. A codon chart shows which codons code for which amino acids.

Suppose a DNA mutation led to a change in a single mRNA codon. Now suppose this codon changed from GCC to GCG. By looking at the codon chart, you can see that both of these codons code for the amino acid alanine. So even though the DNA and mRNA have changed, there is no change in the protein!

This chart shows the amino acids coded for by each of the 64 possible mRNA codons. To find which amino acid the codon CAA codes for, follow these steps. (1) Look on the left side of the chart to find the large row of codons that begin with C. (2) Move across this row until you get to the column of codons whose second base is A. (3) Move down this column until you get to the row of codons whose third base is A. The codon CAA codes for the amino acid glutamine.
When we studied DNA in class, we learned about genetics. My teacher kept using the word allele. What’s an allele?

Let's start with some background information. For each of your inherited traits, you inherit one gene from your mother and one from your father, which means you have two copies of most genes. (The only exception involves sex chromosomes in males.)

Genes can have different forms called alleles. For example, the genes that determine flower color in pea plants have two alleles: one for purple (called P) and one for white (called p). A pea plant could inherit two purple alleles (PP), two white alleles (pp), or one of each (Pp).

O.K., a plant with two purple alleles would have purple flowers, and a plant with two white alleles would have white flowers. But what about a plant with one purple allele and one white allele?

A pea plant with one of each type of allele (Pp) would have purple flowers. This is because the purple allele (P) is dominant. A dominant allele is one that is expressed (or visible) in an organism even if the organism has only one copy of it. The white allele (p) is recessive. A recessive allele is one that is expressed in an organism only if the organism has two copies of it. Pp is the plant's genotype, and purple flowers is the plant's phenotype.

What's the difference between genotype and phenotype?

The genotype tells you which alleles the organism has. And the phenotype tells you which form of the trait is expressed in the organism.

How can you use genetics to make predictions?

Scientists might use a tool called a Punnett square. A Punnett square is used to predict the possible genotypes and phenotypes of offspring. Let's continue to use the example of pea plants. Suppose we cross two plants with purple flowers and genotypes of Pp. The Punnett square below allows us to make predictions about their offspring.

**Punnett Square**

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>PP</td>
<td>Pp</td>
</tr>
<tr>
<td>p</td>
<td>Pp</td>
<td>pp</td>
</tr>
</tbody>
</table>

The alleles of one parent are put along the top of the Punnett square. The alleles of the second parent are put along the left side of the Punnett square. The boxes in the Punnett square are filled in with combinations of the parents' alleles. These combinations are the possible genotypes of the offspring.
Are all genetics problems so simple?

Punnett squares are very basic tools for scientists to use when predicting traits that are simple. But genetics can be very complicated. For instance, humans have 23 pairs of chromosomes and about 30,000 different genes. You should always remember that there are two alleles for each gene, which allows for millions of different combinations. Also, one pair of chromosomes, the sex chromosomes, determine if you are a male or a female, as well as controlling some other traits. The other 22 pairs, the autosomal chromosomes, have little influence on your gender but control most of your other traits. When you consider all of these factors and many more that we are discovering every day, you can see that genetics can be very complicated as well as very important.

Do all plants and animals have the same genes?

No, but some organisms are closely related. Grouping organisms into categories helps scientists find information about living things more easily. For instance, if you know that a cobra is a snake, you know that it has scaly skin and no legs. Biologists group organisms into categories based on how closely related the organisms are. Classifying living things in this way helps biologists study how organisms have changed over time.

What are these categories?

First of all, living things are classified into large groups called kingdoms. Each kingdom is divided into smaller groups, these smaller groups are divided into even smaller groups, and so on. These groups, from largest to smallest, are kingdom, phylum, class, order, family, genus, and species.
How many kingdoms are there? And what’s in them?

Today the most widely used classification system contains six kingdoms. This system has changed over the years to include new information. The following list shows the names and major characteristics of the six kingdoms.

- **Archaebacteria**: This kingdom includes unicellular (one-celled) prokaryotes that often live in extreme environments. Some archaebacteria are **autotrophs** (make their own food), and some are **heterotrophs** (cannot make their own food). Examples of archaebacteria include the bacteria that live in hot springs.

- **Eubacteria**: This kingdom also includes unicellular prokaryotes that may or may not make their own food. However, most eubacteria do not live in extreme environments. Examples of eubacteria include the bacteria that cause strep throat.

- **Protista**: This kingdom includes mostly one-celled eukaryotes. However, there are a few protists that are multicellular. Protists may be autotrophs or heterotrophs. Examples of protists include amoebas, slime molds, and algae.

- **Fungi**: Most fungi are multicellular eukaryotes, although there are a few unicellular fungi. All fungi are heterotrophs with cell walls. Examples of fungi include mushrooms, yeasts, and molds.

- **Plantae**: Plants are multicellular eukaryotes. They have cell walls and specialized tissues and organs. Plants can make their own food, so they are autotrophs. Examples of plants include mosses, ferns, trees, and grasses.

- **Animalia**: Like plants, animals are also multicellular eukaryotes with specialized tissues and organs. However, animals are heterotrophs and lack cell walls. Examples of animals include worms, insects, fish, birds, reptiles, and mammals, including humans.

How do scientists know which group to place organisms in?

Scientists classify organisms based on a variety of characteristics, including genetic makeup, body chemistry, physical structures, and more. Let’s use the transportation of materials as an example of one characteristic that scientists use to classify some types of organisms.

All living things must be able to transport materials such as nutrients and oxygen within their bodies. Organisms that are closely related have similar structures to accomplish transport.

Archaebacteria, eubacteria, and most protists are one-celled organisms. Because they are so small, they do not need to transport materials great distances, and they do not have well-developed transport systems. Some protists, however, have organelles called **contractile vacuoles**. These organelles collect excess water and pump it out of the cell.
Even though many fungi are multicellular, they also lack circulatory systems. Instead, materials within fungi are transported directly from cell to cell. By contrast, many plants have specialized transporting tissues called xylem and phloem. Xylem transports water and minerals throughout a plant, and phloem transports sugars from one part of a plant to another. We'll talk more about this in Objective 3.

Animals have the most-developed transport systems of all. In animals the transport system is called the circulatory system. The circulatory system of many animals includes a heart that pumps blood throughout the body. The blood travels in vessels and carries nutrients, oxygen, and waste products such as carbon dioxide.

Fish have a two-chambered heart, amphibians have a three-chambered heart, and birds and mammals have a four-chambered heart. Fish have a circulatory system with a single loop. Amphibians, birds, and mammals have a circulatory system with two loops, one to the lungs and one to the body.
Objective 2

I think I know the parts of the body systems. Can you tell me what the systems do?

Here’s a review of the body systems and how they function.

- **Circulatory:** The circulatory system transports oxygen and nutrients to cells and carries wastes away from cells.
- **Respiratory:** The respiratory system moves oxygen into the body and carbon dioxide out of the body.
- **Digestive:** The digestive system digests (breaks down) food and absorbs nutrients.
- **Nervous:** The nervous system detects changes outside and inside your body and controls the way your body responds to these changes.
- **Skeletal:** The skeletal system helps you move, protects your internal organs, and gives your body shape and support. It also stores minerals and produces blood cells.
- **Muscular:** The muscular system is responsible for voluntary movements (such as jumping and pointing) and involuntary movements (such as the beating of your heart and the churning of your stomach).
- **Endocrine:** The endocrine system produces chemical messengers called hormones. Some hormones help to maintain homeostasis. Others control development and growth.
- **Integumentary:** The integumentary system (skin) forms a protective barrier around the body. The skin helps prevent water loss and control body temperature. It also gathers information about your surroundings.
- **Immune:** The immune system protects the body from infection.
- **Lymphatic:** The lymphatic system takes fluid from the spaces between cells and returns it to the circulatory system. It also filters bacteria and other microorganisms from this fluid.
- **Reproductive:** In males the reproductive system produces sperm, and in females the reproductive system produces eggs.
- **Excretory:** The excretory system removes wastes from the body and helps maintain homeostasis.

I don’t understand homeostasis. Can you explain it?

When you think of *homeostasis*, think of Goldilocks. Goldilocks couldn’t have her porridge too hot or too cold. Her bed couldn’t be too hard or too soft. Everything had to be “just right.”
What does a fairy tale have to do with science?

Cells are much the same. For example, the fluid that surrounds them can't be too hot, too salty, or too acidic, or they'll die. Everything has to be just right in order for them to survive. Now, here's where homeostasis comes in. Homeostasis refers to an organism's ability to maintain a stable internal state. In other words, homeostasis is the ability of an organism to keep everything just right for its cells.

But how do organisms keep things “just right”?

There are many chemical reactions and physical processes that organisms use to maintain homeostasis. Let's look at one example involving osmosis. Osmosis is the diffusion of water across a semipermeable membrane such as a cell membrane. A semipermeable membrane is one that only certain types of molecules can cross. For example, water can cross a cell membrane, but many other substances can't.

Cell membranes can do this because they are made of two layers of lipids called a lipid bilayer. Each layer of lipid has a polar side and a nonpolar side. In a lipid bilayer, the nonpolar sides are facing each other, and the polar sides are on the outside. The nonpolar interior of the cell membrane prevents ions and large uncharged molecules, such as sugar, from passing through the membrane. However, floating in the cell membrane are transport proteins that will allow certain molecules, even large molecules, to enter the cell. We'll talk more about polar molecules in Objective 4.

Osmosis

The net direction of osmosis through a cell membrane depends on the concentration of dissolved substances inside and outside the cell. Osmosis involves the movement of water from the side with a lower concentration of dissolved substances (higher concentration of water) to the side with a higher concentration of dissolved substances (lower concentration of water).

As the diagram shows, osmosis helps keep the concentration of dissolved substances inside and outside the body's cells nearly the same. There is more dissolved substance inside the cell than outside the cell. So, over time water will move into the cell.
**Objective 2**

**Now It’s Your Turn**

After you answer the practice questions, you can check your answers to see how you did. If you chose the wrong answer to a question, carefully read the answer explanation to find out why your answer is incorrect. Then read the explanation for the correct answer.

**Question 8**

What is one reason that dehydrated patients are given intravenous (IV) solutions of pure water with a small amount of dissolved salt rather than just pure water?

A. To help prevent cells from shriveling because of the pressure caused by osmosis
B. So that the pressure caused by osmosis will cause a net movement of salt into cells
C. So that dissolved substances will be transported across cell membranes and out of cells
D. To help keep the concentrations of dissolved substances inside and outside the cells equal

**Answer Key: page 334**

**Question 9**

As an athlete is running a 5-kilometer race, her cells need more oxygen. Which change will help her body meet the increased demand for oxygen?

A. Her heart beating more quickly
B. Her pancreas releasing more insulin
C. Her breathing becoming more shallow
D. Her sweat glands becoming more active

**Answer Key: page 334**

**Question 10**

An agricultural scientist wants to develop a variety of corn that will mature rapidly and will produce high yields. Which genotypes should the scientist cross to produce the most plants with the desired characteristics?

<table>
<thead>
<tr>
<th>Allele</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-yield</td>
<td>H</td>
</tr>
<tr>
<td>High-yield</td>
<td>h</td>
</tr>
<tr>
<td>Rapidly maturing</td>
<td>M</td>
</tr>
<tr>
<td>Slow-growing</td>
<td>m</td>
</tr>
<tr>
<td>Tall</td>
<td>T</td>
</tr>
<tr>
<td>Short</td>
<td>t</td>
</tr>
<tr>
<td>Yellow kernels</td>
<td>Y</td>
</tr>
<tr>
<td>White kernels</td>
<td>y</td>
</tr>
</tbody>
</table>

A. $\text{hmmTtyy} \times \text{hhMMttYy}$
B. $\text{HHmmtttyy} \times \text{hhMMttYy}$
C. $\text{hhMmtttyy} \times \text{HhmmttYY}$
D. $\text{HHmmtttyy} \times \text{hhMmttYy}$

**Answer Key: page 334**
Question 11

<table>
<thead>
<tr>
<th>DNA</th>
<th>mRNA</th>
<th>Phenotype</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>C-T-T</td>
<td>G-A-A</td>
</tr>
<tr>
<td>Mutation 1 (sickle-cell mutation)</td>
<td>C-A-T</td>
<td>G-U-A</td>
</tr>
<tr>
<td>Mutation 2</td>
<td>C-T-C</td>
<td>G-A-G</td>
</tr>
</tbody>
</table>

What is one possible reason that Mutation 2 leads to the production of normal blood cells rather than sickle-shaped blood cells?

A The mRNA codons GAA and GAG both code for the same amino acid.
B The mRNA codon GAG acts as a stop signal rather than coding for an amino acid.
C The mRNA codon GAG is unreadable and is therefore skipped over during protein synthesis.
D The mRNA codon GAG corresponds to the tRNA molecule that can carry more than one amino acid.
**Question 12**

Histamine is a polar chemical that can lead to an allergic response when it is released by the body's immune system. An antihistamine is a drug that can help prevent the allergic reactions associated with histamine. An antihistamine is a similar molecule to histamine in size, shape, and polarity. How does an antihistamine most likely prevent the effects of histamine?

A. It increases the diffusion of histamine across the membranes of target cells.
B. It binds to histamine receptors on the surfaces of target cells.
C. It causes target cells to increase production of histamine receptors.
D. It blocks histamine receptors found in the cytoplasm of target cells.

**Question 13**

Duchenne muscular dystrophy is a genetic disorder. It results from a mutation in the gene that codes for a protein necessary for muscle strength. A geneticist prepared a pedigree for a family in which the disorder is present in some members. According to this information, what type of allele is responsible for Duchenne muscular dystrophy?

A. Autosomal recessive
B. Autosomal dominant
C. Sex-linked recessive
D. Sex-linked dominant

**Key**

- Male
- Female
- Carries the mutated gene; does not have the disorder
- Carries the mutated gene; has the disorder
Question 14

When a black mouse that is homozygous for coat color (BB) is crossed with a white mouse that is homozygous for coat color (bb), all of the F₁ generation offspring have black coats.

What are the expected genotypes and phenotypes of coat color in the F₂ generation?

A  All F₂ mice have BB genotypes and black phenotypes.
B  All F₂ mice have bb genotypes and white phenotypes.
C  The genotypes of the F₂ mice are 25% BB, 50% Bb, and 25% bb. The phenotypes are 75% black and 25% white.
D  The genotypes of the F₂ mice are 50% BB and 50% bb. The phenotypes are 50% black and 50% white.
From your studies in biology, you should be able to demonstrate the interdependence of organisms and the environment.

Wait a minute! What am I supposed to demonstrate?

Organisms don’t live in isolation. All living things depend on their environment and other organisms for survival. In other words, every living thing is part of an ecosystem. You need to be able to show that you understand this dependence of organisms on one another.

What’s an ecosystem?

An ecosystem is an interactive system that includes all parts of the physical environment (abiotic factors such as temperature, soil, and weather). It also includes the entire community of organisms that live there (biotic factors such as plants, animals, fungi, and bacteria).

Although it’s easy to think of ourselves as separate from the environment, we are actually part of an ecosystem. Humans often see themselves as independent of nature, but nothing could be further from the truth. We depend on nature for basic resources such as food and shelter, clean air, and clean water. Life can exist only within an ecosystem.

So some animals eat other animals; what’s the big deal?

The flow of energy through an ecosystem determines what kinds of organisms live there and how many organisms the ecosystem can support. This is called the carrying capacity of the ecosystem. Many organisms obtain the matter and energy that they need to survive by eating other organisms. I don’t know how you feel, but surviving is a big deal to me!

Eating another animal is just one of the many ways that matter is cycled through an ecosystem.

Ecosystems can be studied by looking at how energy flows through different feeding levels (for example, producers, consumers, and decomposers). These feeding relationships are also called trophic levels. Consider the following important concepts about the flow of energy in an ecosystem:

- Organisms contain chemical energy that is stored in organic molecules, such as sugars, starches, and proteins.
- Energy flows through ecosystems in food chains and food webs every time an organism eats another organism.
● Energy flows in one direction only: from the sun to producers and then to consumers. Energy cannot be recycled. Most energy is released as heat. This means that it cannot be used to do work.

**Food Chain in a Freshwater Stream**

The organisms in each higher level can store only about 10 percent of the energy available in an organism that it eats. About 90 percent of the energy consumed is used to maintain the organism's life functions (metabolism) or lost as undigested food and waste heat. Only about 10 percent is used to make new cells.

Remember that a food chain shows how populations of organisms relate to populations of other organisms.

**What's a population?**

A population is a group of individual organisms that belong to the same species, live in the same general area, and breed with other individuals in the group. Over time the genetic makeup of a population can change in order to adapt to the changes in the environment. This is called evolution.

**Evolution and natural selection are kind of the same thing, right?**

Many students are confused about the difference between evolution and natural selection. To evolve means to change over time. Stars evolve, languages evolve, and in biology, living things evolve. Evolution is the change in the genetic makeup of populations and species over many generations.

**But what about natural selection?**

It's kind of like cause and effect. Natural selection is the cause, and evolution is the effect. Let me explain how the process of natural selection causes change within populations of organisms.

Organisms vary because of differences in inherited characteristics, or traits. We talked about that in Objective 2. Individuals with physical and behavioral traits that are better adapted to an environment are more likely to survive, reproduce, and pass those traits on to their offspring. This process is sometimes called survival of the fittest.

As those individuals with favorable traits in a population produce more offspring, the favorable traits become more common in the population. Over many generations the genetic makeup of a population can evolve.
The environment puts pressure on all organisms. Those that are adapted to survive live to reproduce. Over time the process of natural selection can lead to the evolution of a new species.

**Are you saying that individual organisms don't evolve?**

That’s right. Now you’ve got it! The genetic makeup of an organism doesn’t change during its lifetime, so it can’t evolve. But the genetic makeup of a population changes with each new generation. So populations can evolve.

Sharks (fish) and dolphins (mammals) are not closely related. They’ve evolved bodies of a similar shape because these animals have adapted to the same environment. Both are fast swimmers and can accelerate quickly in water to catch prey.

**Are animals the only things that evolve?**

No, all organisms evolve. Bacteria evolve very quickly. Even fungi and plants evolve.

**Speaking of plants, why are they important?**

Can you imagine a world without plants? I can’t! Think about the foods you eat. Many foods are made from plants in the grass family. These plants produce cereal grains such as wheat, corn, rice, and oats. These grains are used to make foods, such as:

- bread
- tortillas
- spaghetti
- popcorn

But I had a hamburger and a glass of milk for lunch. These foods don’t come from plants!

You’re right, of course. These foods come from animals. But consider what cows eat. They eat plants. And where do you think that hamburger bun came from?

**O.K., so plants provide food. Are they good for anything else?**

Yes! Plants are important for a lot of other reasons:

- Plants play an important role in the carbon cycle. They remove carbon dioxide from the atmosphere and release oxygen. Without plants the oxygen currently in Earth’s atmosphere would last only about 11 years.
- Plants are the dominant group of organisms on land, if biomass is considered. Biomass is the dry mass of tissue and organic matter.
in an ecosystem. It is an important source of energy and the most significant fuel worldwide after coal, oil, and natural gas.

- Plants play an important role in the water cycle and even affect climate. They also help prevent erosion.
- Plants store and recycle essential nutrients through the biosphere. They change energy from the sun into forms of energy that animals can use.
- Plants provide many useful products, such as wood, paper, clothing, and medicines.

We need plants to feed, clothe, and shelter the human population. All animals owe their existence to plants. If there were no plants, there could be no animals, including mammals like us. It's that simple.

**As long as we’re talking about plants, can you explain how they have adapted to their environment?**

Sure! Most plants are composed of three organs that enable them to adapt to life on land: *roots, stems*, and *leaves*.

Roots have three primary functions:

- Absorbing water and minerals
- Anchoring a plant firmly in the ground and providing support
- Producing growth hormones that regulate plant development

The stem provides a supporting framework for the leaves and branches. A plant stem also contains a network of *phloem* and *xylem*. This system of vascular tissue is like a series of tiny pipes that transport water and nutrients throughout a plant, much like the plumbing system in your school.

Leaves are the main organs of photosynthesis in plants. They intercept sunlight and capture carbon dioxide from the atmosphere, manufacture food (glucose), and release water vapor and oxygen.

It is important to remember that many plants have evolved specialized roots, stems, and leaves that help the plant adapt to its environment. These include adaptations for protection and defense, water and nutrient storage, the trapping and digesting of insects, and vegetative (asexual) reproduction. The prickly pear cactus is an example of a plant that has evolved a specialized stem and highly modified leaves.

**You mentioned phloem and xylem. I have trouble remembering what these words mean. I always seem to mix them up! Can you help?**

I used to have trouble with this, too, but I learned a few tricks to help me remember. Phloem is a specialized tissue that carries food (carbohydrates dissolved in water) from the locations where it is made to other cells in the plant. Phloem consists of relatively soft-walled live cells.
To remember what phloem conducts, just remember what the phloem carries: the plant’s food. Phloem and food both start with an F sound. Xylem is a specialized tissue that carries water and dissolved minerals from the roots to other cells in the plant. Mature xylem consists of hard-walled dead cells.

So enough about plants. Earlier you mentioned bacteria. Why do I need to know about something I can’t even see?

It’s true that bacteria are so small that you need a compound light microscope to see them. Viruses are even smaller. Most can’t be observed with a compound light microscope.

The last cold that you caught was probably caused by a virus. Viruses cause many serious diseases in humans, such as chicken pox, the flu, measles, AIDS, and certain cancers. Some viruses can even cause birth defects.

Are all viruses bad?

No, most are harmless to humans. In fact, certain viruses have even been used to cure bacterial infections in which the bacteria are resistant to all known antibiotics. So viruses can save the day—I mean, the patient!

Does this mean that bacteria are bad news?

Yes and no. Some bacteria do cause problems for people. You can blame bacteria if you’ve ever had a tooth cavity or suffered from acne. Bacteria are also responsible for spoiled food. And, of course, bacteria cause many serious human diseases, such as food poisoning, Lyme disease, and strep throat.
What did you mean by “yes and no”?
I’m glad you asked! Most bacteria are actually beneficial.

- Bacteria are important decomposers. They help recycle essential nutrients through the biosphere.
- Helpful bacteria live in the human digestive tract. We couldn’t survive without them. These bacteria promote good health and the absorption of nutrients.
- Bacteria are used to produce foods such as yogurt, cheese, and pickles.
- Humans use genetically engineered bacteria to produce drugs such as human insulin.
- Bacteria are also used to make industrial chemicals, harvest metals from ore, treat raw sewage, and clean up polluted water.

So you see, bacteria really are important. Plant and animal life, including humans, cannot live without Earth’s most primitive and abundant life-form.

If viruses and bacteria both cause disease, what makes them different?
Good question. Bacteria are single-celled organisms. Remember, cells are the basic structure of all living things. All cells contain specialized parts that perform specific functions.

Viruses lack the organization and cellular structure that bacteria possess. Viruses cannot reproduce independently; they need a host cell. For these reasons, viruses are not considered living organisms.

The word virus means “poison” in Latin. Viruses are smaller than bacteria. They range in size from 0.005 to 0.35 micrometers.
Objective 3

Now It’s Your Turn

After you answer the practice questions, you can check your answers to see how you did. If you chose the wrong answer to a question, carefully read the answer explanation to find out why your answer is incorrect. Then read the explanation for the correct answer.

Question 15

Natural selection has most likely favored the shell color of the common purple snail as a response to —

A predation  
B competition  
C average water temperature  
D jellyfish population density

Question 16

A person who is taking antibiotics benefits from eating yogurt that contains live and active bacterial cultures because the bacteria in yogurt —

A release enzymes that prevent the reproduction of viruses  
B may aid antibiotics by eating harmful bacteria in the human digestive tract  
C may restore the normal community of bacteria living in the human digestive tract  
D are a major source of dietary fiber, which helps provide the energy needed to fight an infection

The common purple snail (Janthina janthina) feeds on jellyfish. This snail spends its entire life floating upside down in the open ocean, suspended just below the surface by a raft of air bubbles. The shell has a distinctive two-tone violet color. The base, which is directed toward the surface, is deep violet in color. The top, which is directed downward, is a lighter shade of purple. Viewed from above the water’s surface, the shell blends in with the dark blue of the deep sea. Viewed from below, the shell is difficult to see against a light-blue sky.

After you answer the practice questions, you can check your answers to see how you did. If you chose the wrong answer to a question, carefully read the answer explanation to find out why your answer is incorrect. Then read the explanation for the correct answer.
Which statement about the evolutionary history of jawed fishes is supported by the diagram?

A. Jawless fishes became extinct after jawed fishes evolved.
B. The first amphibians were direct descendants of lungfish.
C. Ray-finned and lobe-finned fishes have a common ancestor.
D. Sturgeon are more closely related to sharks than to coelacanths.
Question 18
Which seed type will most likely be carried by the wind?

A

B

C

D

Question 19
Tropical rain forests support the most-diverse plant communities on Earth. This biome has developed in regions near the equator that are characterized by abundant precipitation and the absence of freezing temperatures. The consistently warm to hot weather and abundant moisture promote rapid chemical weathering and the decay of organic matter. These processes produce thick, nutrient-poor soils.

Plants play a significant role in tropical rain forests by —

A producing thick soils that promote the decay of organic matter
B preventing erosion and allowing nutrients to accumulate in the soil
C holding most of the available nutrients within their biomass
D providing insulation and trapping heat that contributes to the high annual temperatures
Question 20

Partial Desert Food Web: Joshua Tree National Park

Which organisms are both secondary and tertiary consumers in this partial desert food web?

A  Hawk and snake
B  Lizard and wood rat
C  Termite and hawk
D  Snake and lizard

Answer Key: page 337

Question 21

Tuberculosis, or TB, is a contagious bacterial disease that usually occurs as an infection of the lungs. The symptoms of this disease include persistent coughing, fever, fatigue, night sweats, and unexplained weight loss. TB can be treated with antibiotics. Tuberculosis is most likely transmitted —

A  by mosquito bites
B  by blood transfusions
C  through the air
D  through water

Answer Key: page 337
Objective 4

The student will demonstrate an understanding of the structures and properties of matter.

From your studies in chemistry, you should be able to demonstrate an understanding of the structures and properties of matter.

Structures and properties of matter? What’s that?

*Matter* is anything that has mass and takes up space. That includes your pencil, the air you breathe, and even you! All matter is made up of elements or compounds. You need to be able to show that you know what matter is made of and what some of its physical and chemical properties are.

What's an element?

*Elements* are the building blocks of matter. They cannot be broken down into simpler substances by a chemical reaction. Elements are made up of *atoms*.

Model of an Atom

The nucleus of an atom is made up of positively charged protons and neutral neutrons. A cloud of negatively charged electrons surrounds the nucleus of an atom.
The atoms of different elements have different numbers of protons. For example, all oxygen atoms have eight protons, while all nitrogen atoms have seven. Ninety-two elements exist naturally on Earth, and about twenty more have been made in laboratories.

**That’s more than 100 different elements! How am I supposed to keep track of all of them?**

Lucky for you, scientists came up with the periodic table! The periodic table groups elements with similar properties together. Take a look at the periodic table on page 249 of this book.

**What are all those numbers and letters on the periodic table?**

The elements in this periodic table are arranged in order of increasing *atomic number*. The atomic number is equal to the number of protons in each atom of an element. The atomic number is the number listed above the element symbol in each square of this periodic table. For example, this periodic table shows that the atomic number of carbon is 6. Therefore, all carbon atoms have 6 protons.

The periodic table also shows the chemical symbol for each element. The chemical symbol is a one- or two-letter abbreviation. For example, the chemical symbol for carbon is C.

The number listed below the element symbol in each square of this periodic table is the element’s *atomic mass*. The atomic mass of an element is the average mass of one atom measured in atomic mass units (amu). An atomic mass unit is approximately equal to the mass of one proton or one neutron.

The atomic mass of a single atom is approximately equal to the number of protons plus the number of neutrons. So, a nitrogen atom with 7 protons, 7 neutrons, and 7 electrons has an atomic mass of about 14 amu. Because the number of electrons equals the number of protons, the atom as a whole has no charge. Neutrons have no charge and do not affect the charge of the atom.
But how can the periodic table help me make sense of the different elements?

Each column in the periodic table is called a group. The elements in each group have similar properties, such as the number of valence electrons. As a result, metals, nonmetals, and metalloids are clustered together in certain parts of the table.

Notice the heavy bold line on the right half of the periodic table. Metals are found on the left side of this line. Metals are usually shiny solids that can be pounded into sheets. They are also good conductors of heat and electricity.

Nonmetals are found on the right side of the heavy bold line on the periodic table. Many nonmetals are gases or liquids under normal conditions. Solid nonmetals are brittle and dull. They are poor conductors of heat and electricity.

Most of the elements that border the heavy bold line are metalloids. Metalloids are located between the metals and nonmetals, and they have properties similar to both groups. Metalloids often act as semiconductors. This means that they normally do not conduct electricity but can be made to do so under certain conditions.

You mentioned valence electrons. What are those?

Much of chemistry is about the movement of electrons. In a chemical reaction, bonds between atoms are formed or broken. These bonds involve the transfer or sharing of electrons between atoms.

Valence electrons are the outermost electrons in the electron cloud surrounding an atom's nucleus. Different elements have different numbers of valence electrons. Because valence electrons are the farthest from the nucleus, they can move from one atom to another much more easily and can participate in bonding.
So how does the periodic table help me figure out how many valence electrons an atom has?

All the elements in some groups of the periodic table have the same number of valence electrons. There is a pattern that can be observed.

<table>
<thead>
<tr>
<th>Group numbers</th>
<th>1</th>
<th>2</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of valence electrons</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Elements in Group 1 have one valence electron, and elements in Group 2 have two valence electrons. For Groups 13 through 18, the number of valence electrons is equal to the group number minus 10. The exception to this rule is helium (He). Helium is in Group 18. Helium atoms have only two electrons, so they have two valence electrons rather than eight.

Elements can be solids, liquids, and gases, right?

Yes. A state of matter is the form in which a substance exists. Solid, liquid, and gas are states of matter.

The atoms in a solid are packed closely together. As a result, a solid has a definite volume and shape. An iron nail is an example of a solid.

The atoms in a liquid are close together but can flow over one another. A liquid has a definite volume but takes the shape of its container. For example, when you pour water from a pitcher into a glass, the shape of the water changes, but the volume of the water stays the same.

The atoms in a gas are spread apart and can move throughout their container. A gas doesn’t have a definite volume or shape. When you pop a balloon, for instance, the air inside it doesn’t keep the shape and volume of the balloon. Instead, the air spreads out into the room.

**States of Matter**

![Solid](image1)

![Liquid](image2)

![Gas](image3)

The spheres in this diagram represent the atoms of a solid, a liquid, and a gas.
Can substances change state?
Yes! Substances can change from one state of matter to another. For example, when ice is heated to its melting point, it changes from solid ice to liquid water. Similarly, when water is heated to its boiling point, it changes from liquid water to water vapor (a gas).

A change in state is a physical change. In a physical change no new substances are produced. Water changes form when it freezes to ice, but water and ice are both H₂O.

Pressure can also affect states of matter. For example, oxygen is a gas under normal conditions, but it can be condensed to a liquid at very high pressures, even if the temperature doesn’t change.

What about a chemical change? What’s that?
A chemical change is one in which new substances are formed. The atoms of the original substances are rearranged to form the new substances. The new substances often have properties that are very different from those of the original substances.

Under certain conditions hydrogen gas and oxygen gas can react to form water. This reaction is a chemical change. The atoms of hydrogen gas and oxygen gas have been rearranged to form water molecules. Water has properties very different from those of either hydrogen or oxygen. For example, water is a liquid at room temperature and normal pressure, but hydrogen and oxygen are both gases under these conditions.

How can I spot a chemical change?
One sign of a chemical change, or chemical reaction, is a change in temperature. Some chemical changes produce heat; others absorb heat from their surroundings. For example, when hydrogen and oxygen gas react to form water, a large amount of heat is produced.
The exothermic reaction of liquid hydrogen and liquid oxygen is used to help launch a space shuttle.

Another sign of a chemical change is a change in color. When you burn a piece of white paper, the color changes from white to black. Burning is a chemical change.

The production of a *precipitate* or a gas also signals a chemical change. If you mix baking soda and vinegar, for example, gas bubbles are produced. The baking soda and vinegar have reacted to form carbon dioxide gas—a new substance.
It is important to remember that if you observe only one of these signs, a chemical reaction may not necessarily have taken place. When water boils, for instance, gas bubbles appear in the water, but no chemical change occurs, only a physical change. The only sure sign of a chemical change is the production of a new substance.

**Can you help me with chemical equations?**

No problem! A chemical equation is a way of writing out what happens during a chemical change. For example, a chemical equation can be used to describe the reaction of hydrogen gas with oxygen gas.

\[ \text{H}_2 + \text{O}_2 \rightarrow \text{H}_2\text{O} \quad \text{(unbalanced)} \]

A chemical equation contains a lot of information. The left side of the equation shows what you started with (the reactants). In this example the reactants are hydrogen gas \((\text{H}_2)\) and oxygen gas \((\text{O}_2)\). The right side of the equation shows what you ended up with (the products). In this example, the product is water \((\text{H}_2\text{O})\).

This chemical equation, however, isn’t quite finished. To complete the equation, we need to balance it.

**Balance it? How do we do that?**

When we balance an equation, we make sure that the left side of the equation has the same number and types of atoms as the right side of the equation. We need to balance the equation to show that it follows the law of conservation of mass.

**The law of what?**

The law of conservation of mass states that matter cannot be created or destroyed in a chemical reaction. All the atoms of the reactants in a chemical reaction are still present in the products; they’ve just been rearranged. We need to balance the equation to show this.

**How do we do that exactly?**

First let’s look at the hydrogen. The left side of the equation has two atoms of hydrogen, and so does the right side, so hydrogen is balanced.

\[ \text{H}_2 + \text{O}_2 \rightarrow \text{H}_2\text{O} \quad \text{(unbalanced)} \]

Next look at the oxygen. The left side of the equation has two atoms of oxygen, but the right side has only one atom of oxygen. That means we’ll have to add a “2” in front of the “\(\text{H}_2\text{O}\)” This number is called the coefficient. When balancing a chemical equation, you should change only the coefficients. The symbol \(2\text{H}_2\text{O}\) means “two molecules of water.”

\[ \text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O} \quad \text{(unbalanced)} \]
Oops! Now there are two hydrogen atoms on the left side and four on the right. Let's fix the hydrogen on the left so that the atoms balance. Recheck to make sure everything follows the law now. The left side of the equation has four hydrogen atoms and two oxygen atoms. So does the right side. Everything balances!

Tell me about solutions. What are they?
A solution is a mixture in which one substance dissolves in another. When a substance dissolves, it breaks up into tiny particles that spread evenly throughout the mixture. The substances in a solution are distributed evenly.

A solution has two parts: the solute and the solvent. The solute is the substance that dissolves, and the solvent is the substance that the solute dissolves in. In a solution of salt water, salt is the solute, and water is the solvent.

Are all solutions liquids?
No. Many types of solutions are possible. Here are a few examples.

### Examples of Solutions

<table>
<thead>
<tr>
<th>Combinations of States</th>
<th>Solute</th>
<th>Solvent</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas–Gas</td>
<td>Oxygen and other gases</td>
<td>Nitrogen</td>
<td>Air</td>
</tr>
<tr>
<td>Gas–Liquid</td>
<td>Carbon dioxide</td>
<td>Water</td>
<td>Soda water</td>
</tr>
<tr>
<td>Liquid–Liquid</td>
<td>Acetic acid</td>
<td>Water</td>
<td>Vinegar</td>
</tr>
<tr>
<td>Solid–Liquid</td>
<td>Salt</td>
<td>Water</td>
<td>Salt water</td>
</tr>
<tr>
<td>Solid–Solid</td>
<td>Zinc</td>
<td>Copper</td>
<td>Brass</td>
</tr>
</tbody>
</table>
Objective 4

I know that water is a polar solvent. What does that mean?
I'll explain in a minute. But first let me talk about the sharing of electrons in molecules. Atoms on the right side of the periodic table (such as oxygen) tend to pull more strongly on the electrons in a covalent bond than atoms of the left side of the periodic table (such as hydrogen).

In a water molecule the oxygen atom tends to pull the shared electrons away from the hydrogen atoms, so the oxygen end of a water molecule has a partial negative charge and the hydrogen end has a partial positive charge. Molecules with a slightly negative end and a slightly positive end are called polar molecules.

Water Molecule

The unequal sharing of electrons in a water molecule gives the molecule a slightly negative end and a slightly positive end.

What does this have to do with water in solutions?

Because water is polar, it tends to dissolve other polar compounds. For example, many ionic compounds dissolve in water. When ionic compounds dissolve, they break up into positive and negative ions.

The negative end of water molecules is attracted to the positive ions, and the positive end of water molecules is attracted to negative ions. Each solute ion becomes surrounded by a “shell” of water molecules. This helps keep the solute ions in solution.
When sodium chloride (NaCl) dissolves in water, it breaks up into sodium ions (Na\(^+\)) and chloride ions (Cl\(^-\)). The negative end of water molecules is attracted to sodium ions, and the positive end of water molecules is attracted to chloride ions.

**What are some other properties of water?**

*Cohesion* is another property of water. Cohesion is the tendency of water molecules to stick together. It is caused by the attraction of the positive end of one water molecule for the negative end of another.

To see cohesion in action, gently lay a paper clip on top of a glass of water. As long as you don’t break the surface, the paper clip will remain on top of the water. The attraction of the water molecules to one another is strong enough to keep the paper clip from sinking.

Because water molecules are polar, they also have a tendency to stick to other polar substances. This property is called *adhesion*. For example, glass can carry a partial charge along its surface. That’s why rain droplets stick to the windshield of a car.
The polar nature of water also causes ice to float. To understand why, you first have to understand density and buoyancy.

**O.K., what’s density?**

Density is a measure of a substance’s mass per unit of volume. A really dense object has much more mass in a given space than an object that isn’t very dense.

\[ \text{Density} = \frac{\text{mass}}{\text{volume}} \]

For example, aluminum isn’t very dense. That’s why empty aluminum soda cans don’t have much mass. If soda cans were made out of a dense metal, such as gold or lead, they would be much heavier than aluminum cans, even if they were the same size.

Suppose you are going to buy a gold necklace at a discount jeweler. The clerk claims that the necklace is made of pure gold. How could you tell if he is telling the truth? Calculate the density of the necklace and compare it to the density of pure gold.

**How do I solve density problems?**

You can calculate an object’s density by dividing its mass by its volume.

Here’s an example. A graduated cylinder containing 20 milliliters of mineral oil has a mass of 98.2 grams. The mass of the empty cylinder is 79.8 grams. What is the density of the mineral oil?

To find the density, we need to know the mass and the volume of the mineral oil. The volume is 20 milliliters. The mass of the oil is equal to the mass of the filled cylinder (98.2 g) minus the mass of the empty cylinder (79.8 g).

\[
\text{Mass of oil} = 98.2 \text{ g} - 79.8 \text{ g} = 18.4 \text{ g}
\]

Now substitute the mass and volume of the oil into the density formula.

\[
\text{Density} = \frac{\text{mass}}{\text{volume}}
\]

\[
D = \frac{m}{v}
\]

\[
D = \frac{18.4 \text{ g}}{20 \text{ mL}}
\]

\[
D = \frac{0.92 \text{ g}}{\text{mL}}
\]

So the density of the mineral oil is 0.92 g/mL.
What about buoyancy?

When an object is placed in water, the water exerts a force on all sides of the object. This force increases with depth, so the force at the top of the object is lower than the force at the bottom of the object. This means that the overall direction of the force is upward. This upward force is called the **buoyant force**.

When the buoyant force pushing up on the object is greater than the force of gravity pulling down on the object, the object rises to the surface. If the buoyant force is less than the force of gravity, the object sinks to the bottom.

![Diagram of buoyancy](image)

When wood floats, the buoyant force on the wood is greater than the force of gravity on the wood.

**How do we know which is greater, the force of gravity or the buoyant force?**

We can determine which force is greater by comparing the density of the object to the density of water. If the density of the object is greater than the density of water, the force of gravity on the object will be greater than the buoyant force, and the object will sink.

If the density of the object is less than the density of water, the force of gravity on the object will be less than the buoyant force, and the object will float. For example, wood with a density of 0.4 gram per cubic centimeter will float on water, which has a density of 1.0 gram per cubic centimeter.
Why does ice float in water? Shouldn’t the solid form of water be denser than its liquid form?

When most substances freeze, the molecules making up the substance get closer together. That means that the density of the solid is greater than the density of the liquid form. For example, solid wax is denser than liquid wax, so solid wax would not float on liquid wax.

But water is different. When water freezes, the polar molecules line up, positive to negative, forming a crystal pattern. This pattern causes the molecules to spread apart. This means that ice is less dense than water. Therefore, it floats.

Structure of Ice and Water

Because ice has a crystalline structure, it has a more orderly arrangement of molecules than liquid water does. This orderly arrangement keeps the molecules in ice from packing together as closely as the molecules in liquid water do.

Because ice is less dense than water, lakes begin to freeze from the top down. The layer of ice that forms on the surface of a lake helps shield the water underneath from cold air temperatures. This tends to keep lakes from freezing solid and killing the organisms that live there.
Now It’s Your Turn

After you answer the practice questions, you can check your answers to see how you did. If you chose the wrong answer to a question, carefully read the answer explanation to find out why your answer is incorrect. Then read the explanation for the correct answer.

Question 22

A student measures the mass of an empty graduated cylinder as 87.76 grams. The student then pours a liquid into the cylinder and places it on the scale. According to the student’s measurements, what is the density of the liquid in grams per milliliter? Record and bubble in your answer.

Question 23

Why is a ship with a hollow steel hull able to float in seawater?

A The density of steel is greater than the density of seawater.
B The buoyant force on the ship is less than the weight of the ship.
C The ship displaces a volume of seawater that weighs more than the ship.
D The buoyant force on the ship is less than the weight of the seawater displaced by the ship.

Question 24

Baking soda consists of the compound sodium bicarbonate (NaHCO₃). When baking soda is heated, sodium carbonate (Na₂CO₃) is produced. Two hundred grams (200 g) of sodium bicarbonate is placed in a test tube and heated with a Bunsen burner. After the reaction is complete, the only substance remaining in the test tube is 126 grams of sodium carbonate. Which best explains why this reaction does not violate the law of conservation of mass?

A Sodium bicarbonate has a greater molecular mass than sodium carbonate.
B The reaction has one or more products that leave the test tube in the form of a gas.
C The high heat of the Bunsen burner destroys some of the atoms in the sodium bicarbonate.
D In the balanced chemical equation, the mass of the reactants is greater than the mass of the products.
**Objective 4**

**Question 25**
A teacher has prepared a saturated solution of potassium nitrate (KNO₃) in 100 grams of water heated to 60°C. About how many grams of potassium nitrate will have settled out of the solution once it reaches a room temperature of 25°C?

A 30 g  
B 65 g  
C 95 g  
D 145 g

**Answer Key: page 339**

**Question 26**
Each beaker shown below contains one liter of water. One hundred grams of sugar is added to each beaker. In which beaker will the sugar dissolve the fastest?

A  
B  
C  
D

**Answer Key: page 339**
Question 27
Thermal pollution occurs when human activities cause the temperature of lakes or rivers to rise. Why are fish most likely to be harmed by long-term thermal pollution of the lake in which they live?

A  The solubility of oxygen in the lake will decrease.
B  The solubility of carbon dioxide in the lake will increase.
C  The solubility of potassium fertilizers, such as KCl, will decrease in the lake.
D  The solubility of quartz crystals (SiO₂) will increase in the lake.

Question 28
The unbalanced chemical equation shows the reaction that occurs when a piece of aluminum foil is placed in a solution of water and copper sulfate.

\[
\text{Al(s)} + \text{CuSO}_4(aq) \rightarrow \text{Al}_2(SO_4)_3(aq) + \text{Cu(s)}
\]

Which set of coefficients balances the chemical equation?

A  1, 3, 1, 3
B  2, 1, 1, 1
C  2, 3, 1, 1
D  2, 3, 1, 3
From your studies in physics, you should be able to demonstrate an understanding of motion, forces, and energy.

What’s so important about motion, forces, and energy?
Look around you. What do you see that’s moving? A breeze blows across the room. Clouds float across the sky. Even your eyes move to read the words on this page. Physicists use the ideas of force and energy to describe motions in the world around us.

How are motion and forces related?
All motion is caused by forces. We can use forces to explain why things move and to predict how they will move. For instance, a roller-coaster designer needs to understand how forces work to create thrilling rides!

There are often many different forces acting on an object. For instance, when you place an apple on a table, gravity pulls the apple down, but the table holds it up. These two forces balance each other. As a result, the apple stays put.

If you pushed the apple off the table, the force of the table would no longer be there to balance the force of gravity. The unbalanced force of gravity would cause the apple to fall to the ground.
When the forces on the apple are balanced, the apple doesn’t move. When the force is unbalanced, the apple falls.

**Didn’t Isaac Newton have something to do with falling apples?**

Yes. Sir Isaac Newton proposed the laws of motion in the seventeenth century. These laws were based on his observations of the world around him. Newton’s laws describe how forces and motion are related. Here they are:

**Newton’s Laws of Motion**

- **First Law:** Any object in motion will stay in motion, and any object at rest will stay at rest, until it is acted on by an unbalanced force. This law is also referred to as the law of inertia.
- **Second Law:** The net force on an object equals the object’s mass multiplied by its acceleration (Force = mass × acceleration).
- **Third Law:** When one object exerts a force on a second object, the second object exerts an equal but opposite force on the first object.

These laws sound a bit complicated at first, but they make sense when you think about them carefully.
If I roll a ball across the floor, the ball eventually stops. But Newton’s first law says that a moving object should keep moving. How does that make sense?

A ball rolling across the floor does eventually stop. But it doesn’t break Newton’s first law. To find out why, look more closely at the law. It states that a moving object should keep moving unless there is an unbalanced force on it. There must be a force acting on the ball that causes it to slow down and stop.

What is that force?

It’s friction! Friction is an unbalanced force that changes the ball’s motion. If there were no friction between the ball and the floor, the ball would keep rolling.

The second law is really just an equation. What’s so important about that?

Newton’s second law ($F = ma$) is a powerful tool for making predictions about motion. For example, if you know how strong a force is and what size mass it acts on, you can predict how fast an object will accelerate.

How can I understand the second law without using math?

It’s easy! First let’s look at force and acceleration. According to Newton’s second law, a greater force produces a greater acceleration. For instance, if you hit a baseball with a bat as hard as you can, it will accelerate more than if you just tap it lightly. The greater the force you exert, the greater the ball’s acceleration. That makes sense, right?

We can also think about mass and acceleration. The smaller the mass, the greater the acceleration will be. If you use the same force to throw a bowling ball and a tennis ball, which goes farther? The tennis ball, of course! It accelerates more because it has less mass than the bowling ball.

Umm . . . could you explain the third law a bit better?

I’m glad you asked! Newton’s third law states that all forces come in pairs. For example, when you bounce a basketball, the ball exerts a force on the floor. The floor also exerts a force back on the ball. That’s why the ball bounces. The floor’s force on the ball is exactly the same size as the ball’s force on the floor. But the two forces act in opposite directions.
Do these pairs of forces cancel each other out?

Good question. You might think that since the forces are equal and opposite, they would cancel each other. Why don’t they? Well, they are acting on different objects. In order to cancel each other, two opposite forces must act on the same object.

For example, suppose that you and I are wearing roller skates and facing each other. If I push on you, you will push back on me with an equal but opposite force (even if you aren’t aware of it). My force on you will cause you to move backward, and your force on me will cause me to move backward. Even though the forces are equal but opposite, they do not cancel out, because one acts on me and one acts on you.

When one skater pushes on the other, the second skater pushes back with an equal but opposite force. There is a net force on each skater, and each rolls backward.
How do I use the formula chart to solve physics problems?

Many ideas in physics, such as Newton's second law, can be written as formulas. You can use these formulas to solve problems and make predictions. Look at the formula chart located on page 248. To use a formula, you need to know what each variable represents. The words on the left side of the formula chart tell you this. For example, look at this formula.

\[ P = \frac{W}{t} \]

The left side of the chart shows that this is the formula for power. \( P \) represents power, \( W \) represents work, and \( t \) represents time.

**After the formulas there is a table titled “Constants/Conversions.” What are these?**

This table gives you extra information you might need to solve problems. For example, constants are values for things that don’t change, such as the speed of light (\( c \)) or the acceleration due to gravity (\( g \)).

The table also shows some common conversions. A conversion shows you how to change from one unit to another. For example, the table shows that 1 newton (\( N \)) is equal to 1 kgm/s\(^2\). You can use this conversion to change the answer to a problem from 8 kgm/s\(^2\) to 8 N.
O.K., I think I’m ready to try a force problem. Can you show me how to do one?

I was hoping you would ask that! Mako sharks have a mass of about 450 kilograms. Let’s calculate the weight of a mako shark in newtons. (Calculating the weight is much easier than trying to get a shark onto a scale!)

Mako Shark

First figure out which formula you should use. Let’s list what we know and what we want to find out. We don’t know much—only the mass of the shark (450 kilograms). We want to find out the weight of the shark. Look at the formula chart on page 248 for a formula that relates weight to mass.

I don’t see one! There isn’t a formula with weight in it. What do we do now?

Now we think! What is weight? Weight is the force of gravity on an object, so weight is a force. Find the formula that relates force to mass.

\[ F = ma \]

Remember this formula? It’s Newton’s second law of motion!

Next, substitute the values you know and solve for what you want to find out.
Wait a minute! What value are we supposed to use for acceleration?

Remember that weight refers to the force of gravity, so the acceleration we should use is the acceleration due to gravity. This value is listed in the constants/conversions table.

\[ g = \text{acceleration due to gravity} = 9.8 \text{ m/s}^2 \]

Okay, we know the mass (450 kg). We know the acceleration (9.8 m/s²). We're ready to use the formula.

\[ F = ma \]
\[ F = 450 \text{ kg} \times 9.8 \text{ m/s}^2 \]
\[ F = 4410 \text{ kgm/s}^2 \]

The force of gravity, or the weight, is 4410 kgm/s². But we want the weight in newtons. The constants/conversions table shows that 1 newton (N) is equal to 1 kgm/s². So, 4410 kgm/s² = 4410 N.

Cool! We used math to solve a physics problem! The shark weighs 4410 newtons. Can you show me how to do a two-part problem—one with two formulas?

Sure! Here's a sample problem: A forklift pushes a crate with a mass of 55 kilograms. The crate accelerates at a rate of 0.5 m/s². If 209 joules of work is done on the crate, how many meters does the crate move?

First, let's write down what we know and what we want to know.

mass = 55 kg  \quad \text{acceleration} = 0.5 \text{ m/s}^2  \quad \text{work} = 209 \text{ J}

distance = ?

Now, look at the formula chart on page 248 to see which equations to use. Which equations include these four variables: mass, acceleration, work, and distance?
The force formula relates mass and acceleration. And the work formula relates work and distance. Can we use these two formulas?

Yes, we can!

\[ F = ma \]

\[ W = Fd \]

We know the mass and the acceleration, so we can use the first equation to solve for force. Then we can substitute force and work into the second equation and solve for the distance.

Let's solve the first equation.

\[ F = ma \]

\[ F = 55 \text{ kg} \times 0.5 \text{ m/s}^2 \]

\[ F = 27.5 \text{ kgm/s}^2 \]

\[ F = 27.5 \text{ N} \]

The force on the crate is 27.5 newtons. We can use this value for force to solve the second equation. First, we'll need to get distance on one side of the equation by itself. Divide both sides of the equation by force.

\[ \frac{W}{F} = d \]

Now substitute the values we know.

\[ \frac{209 \text{ J}}{27.5 \text{ N}} = d \]

\[ \frac{209 \text{ Nm}}{27.5 \text{ N}} = d \]

7.6 m = d

So the crate moves a distance of 7.6 meters.
Wait! In that second step you changed the work from 209 J to 209 Nm. Why did you do that?

I did that so that we would end up with units of meters. The constants/conversions table shows that 1 joule (J) is equal to 1 newton • meter (Nm).

So 209 J = 209 Nm. When we divide 209 Nm by 27.5 N, the units of newtons cancel, leaving us with units of meters, which is what we wanted.

Can we do one more? How about a momentum problem?

A car has a mass of 900 kilograms. The car travels 600 meters in the same direction for 30 seconds at a constant speed. Let’s find the car’s momentum.

First let’s write down what we know and what we want to know.

mass = 900 kg  distance = 600 m  time = 30 s
momentum = ?

Now, look at the formula chart to see which equations to use. We want to find momentum, so we will probably need to use the momentum formula.

\[ \text{Momentum} = \text{mass} \times \text{velocity} \]
\[ p = mv \]

We know the mass, but we don’t know the velocity. We’ll need to calculate the velocity in order to find the momentum.

The only velocity formula in the formula chart is for the velocity of a wave. We need the velocity of a car. What should we do?

Let’s think about velocity. The velocity of an object is its speed in a particular direction. We know that the car isn’t changing direction. We can use speed in place of velocity. And, lucky for us, there’s a formula for speed in the formula chart.

\[ \text{Speed} = \frac{\text{distance}}{\text{time}} \]
\[ s = \frac{d}{t} \]
Let's substitute the values we know and solve the equations. First let's find the car's speed.

\[
\text{Speed} = \frac{\text{distance}}{\text{time}}
\]

\[
s = \frac{d}{t} = \frac{600 \text{ m}}{30 \text{ s}} = 20 \text{ m/s}
\]

The car's speed is 20 m/s. We can use this value for the velocity. Now we're ready to find the momentum.

\[
\text{Momentum} = \text{mass} \times \text{velocity}
\]

\[
p = mv = 900 \text{ kg} \times 20 \text{ m/s} = 18,000 \text{ kgm/s}
\]

The car's momentum is 18,000 kgm/s.

**O.K., enough about motion and forces. What about energy?**

Every day you get energy from the food you eat and then use that energy to study, to play sports, or even just to relax. Energy plays a big part in your life.

All moving objects have energy. For example, a roller coaster climbing a steep hill has energy of motion, or kinetic energy. An object can also have potential energy. A roller coaster sitting at the top of a hill has potential energy because of its height above the ground. This energy is changed into kinetic energy once the roller coaster starts to roll downhill.
What happens to the roller coaster’s energy at the end of the ride? Does it just disappear?

No, it doesn’t! It just changes form. The law of conservation of energy states that energy is neither created nor destroyed. The roller coaster’s energy doesn’t disappear; it just changes into a different form when the roller coaster comes to a stop at the end of the ride.

When the roller coaster slows down, the friction between the wheels of the cars and the rails increases. Friction generates heat. The mechanical energy of the roller coaster is changed into heat energy when it comes to a stop at the end of the ride.

How does energy move from one object to another?

Heat energy always moves from a warmer object to a colder object. There are three ways that heat energy can move: radiation, conduction, and convection. Take a look at this snake basking on a rock.

Three Types of Heat Transfer

Solar energy from the sun travels through space in the form of electromagnetic waves. This energy is transferred to Earth’s surface in the form of heat. Heat transfer by means of electromagnetic waves is called radiation. The sun is not the only source of radiant energy. For example, you can also feel radiation by sitting near a fire in a fireplace.
Another type of heat transfer is *conduction*. Conduction is the transfer of heat within an object or between objects that are touching each other. For example, when the snake's rock absorbs heat through radiation from the sun, some of the heat is transferred to the snake by conduction.

The third kind of heat transfer is *convection*. Convection is the transfer of heat by the movement of a current. A current that transfers heat is called a *convection current*. For example, heat from Earth's surface warms the air closest to the ground. Warm air is less dense than cool air, so warm air rises, and cooler air sinks close to the surface. The rising warm air and the sinking cool air together form a convection current. This current transfers heat within the atmosphere.

**O.K., now I know how heat moves. How about waves? How do they travel?**

A *wave* is a disturbance that transfers energy from one place to another. All waves are produced by some kind of vibration. In a *transverse wave* the vibration is perpendicular to the direction in which the wave travels. For example, if a transverse wave travels from left to right, the medium vibrates up and down.

In a *longitudinal wave* the vibration of the wave is parallel to the direction in which the wave travels. If a longitudinal wave travels from left to right, the medium vibrates left and right as well.

Both waves travel from left to right. But in a transverse wave the medium vibrates up and down, and in a longitudinal wave the medium vibrates back and forth.
How do scientists measure waves?

Scientists measure waves by describing their properties. Some of the properties of waves are wavelength, amplitude, speed, and frequency.

Take a look at the transverse wave below. The wavelength is the distance from one crest (or high point) to the next or from one trough (or low point) to the next. Because wavelength is a distance, it can be measured in meters (m).

The amplitude of the wave is the distance from the resting position to a crest or from the resting position to a trough. Amplitude can also be measured in meters.

![Diagram of a wave showing wavelength, amplitude, crest, and trough.]

The greater the amplitude of a wave, the more energy the wave transfers. If you think about it, this makes sense. A huge ocean wave has a much larger amplitude than a tiny ripple. It also has more energy. A tiny ripple doesn't have enough energy to knock you off a surfboard, but a huge ocean wave might!

What's the difference between speed and frequency?

The speed of a wave is the distance the wave travels in one unit of time. So the speed of a wave can be measured in units of meters per second (m/s).

To measure frequency, you need to figure out how many wavelengths pass a particular point in one unit of time. Frequency is measured in units called hertz (Hz). One hertz is equal to 1 wave per second. For example, if 3 complete wavelengths pass you every second, the frequency is 3 waves per second or 3 hertz.
What about electromagnetic waves? What are they?

Electromagnetic waves include radio waves, microwaves, infrared waves, visible light, ultraviolet rays, and X rays. Without electromagnetic waves, you wouldn't be able to listen to the radio, watch television, use a microwave oven, or even look at a sunset.

Electromagnetic waves are different from other waves because they can travel through a vacuum like space. They don't require a medium as other waves do.

All electromagnetic waves travel through a vacuum at the same speed. This constant speed is often called the speed of light. The speed of light is equal to $3 \times 10^8$ meters per second.

Wait a minute. Isn't there a formula about waves on the formula chart? How do I use it?

The formula chart tells you that the velocity of a wave equals its frequency times its wavelength. Suppose a radio station broadcasts radio waves at a frequency of $1.021 \times 10^8$ hertz. Let's find the wavelength of the waves.

We already know which formula we want to use—the one that relates frequency to wavelength.

$$v = f\lambda$$

Let's rearrange this formula so that we can solve for wavelength. Divide both sides of the equation by frequency.

$$\frac{v}{f} = \lambda$$
Objective 5

But we don’t know the velocity of the wave. How can we solve for the wavelength without the velocity?

Hmm . . . let’s think. We’re talking about radio waves. Radio waves are electromagnetic waves, and electromagnetic waves travel at the speed of light. Because we don’t care what direction the waves are traveling, we can substitute the speed of the waves for the velocity. (Remember that velocity is speed in a particular direction.) The constants/conversions chart tells us that the speed of light is $3 \times 10^8$ m/s.

Let’s substitute the values we know into the formula and solve for wavelength.

$$\text{wavelength} = \frac{\text{velocity of a wave}}{\text{frequency}}$$

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{1.021 \times 10^8 \text{ Hz}}$$

$$\lambda = \frac{300,000,000 \text{ m/s}}{102,100,000 \text{ Hz}}$$

$$\lambda = 2.938 \text{ m}$$

The wavelength of the radio waves is 2.938 meters.
Now It’s Your Turn

After you answer the practice questions, you can check your answers to see how you did. If you chose the wrong answer to a question, carefully read the answer explanation to find out why your answer is incorrect. Then read the explanation for the correct answer.

Question 29

During an earthquake, primary (P) waves travel faster than secondary (S) waves. The difference in arrival time between a P wave and an S wave can be used to determine the distance from a seismograph to the epicenter of the earthquake. A seismograph station records the first P wave of an earthquake at 05:26:00 (hour:minute:second). If the epicenter of the earthquake is 4,000 kilometers from the station, at what time will the station record the first S wave?

A 05:20:30
B 05:30:00
C 05:31:30
D 05:38:30

Question 30

It takes a weight lifter 4.0 seconds to lift a barbell 1.5 meters. He exerts a force on the barbell of 1500 newtons. About how much power does the weight lifter use to lift the barbell?

A 375 watts
B 563 watts
C 2250 watts
D 9000 watts
Objective 5

**Question 31**
During a test of vehicle safety standards, four different vehicles were driven at a test wall at 35 km/h. Which vehicle would most likely hit the wall with the greatest force?

A  A bicycle  
B  A motorcycle  
C  A two-door sports car  
D  A four-door family car

**Question 32**
In which circuit will Bulb 3 remain lit if Bulb 1 burns out?

- **A**
- **B**
- **C**
- **D**

Battery

Bulb

---

Answer Key: page 340

Answer Key: page 340
Question 33
A man standing at a bus stop hears the siren of a parked ambulance. As the ambulance begins moving toward the bus stop, the man hears a change in the sound of the siren.

How do the characteristics of the sound waves change as the ambulance begins moving toward the man?

A The amplitude of the waves increases.
B The wavelength of the waves increases.
C The velocity of the waves increases.
D The frequency increases.

Question 34
A stone is dropped from a bridge and hits the river beneath the bridge 2.30 seconds later. Ignoring the effect of air resistance, what is the stone’s approximate velocity when it hits the river?

A 0.235 m/s
B 4.26 m/s
C 12.1 m/s
D 22.5 m/s

Question 35
In which of the following is the greatest amount of work done?

A Pushing a crate 9.8 meters with a force of 10 newtons
B Pulling a wagon 5.2 meters with a force of 50 newtons
C Pulling a sled 2.3 meters with a force of 90 newtons
D Pushing with a force of 150 newtons on a car that does not move
Cluster 1

Use the information below and your knowledge of science to help you answer questions 36–39.

Partial Compost-Pile Food Web

Note: Organisms not drawn to scale
Question 36
Starting in June, a student observed the animals in a compost pile over a period of one year. In September the student removed as many centipedes from the compost pile as she could find and released them in a park several miles away. At the end of one year, there were many more spiders in the compost pile than had been present the year before. There were also fewer springtails and beetles. Which statement best explains why the populations of springtails and beetles decreased?
A They had less food available to them.
B They had less space available to them.
C They faced increased predation by carnivores.
D They faced increased competition from herbivores.

Question 37
A group of students set up two compost piles (A and B) that each contain the same amount and type of dead organic material. Both piles are sheltered from precipitation. Twice a week the students add water to Compost Pile A but add no water to Compost Pile B. Once a week the students estimate the numbers of different types of organisms in each pile by random sampling.

Which question are the students most likely trying to answer?
A How does the moisture level in a compost pile change over time?
B What types of organic material lead to the most-diverse compost-pile food webs?
C Do more organisms live in compost piles with moist environments or dry environments?
D How does the number of carnivores in a compost pile affect the number of omnivores?

Question 38
A chloroplast is an organelle that absorbs sunlight and uses it to produce sugars. Which part of the compost pile is made up of cells that contain chloroplasts?
A Fungi
B Bacteria
C Sow bugs
D Grass clippings

Question 39
A gardener lifts a bag filled with compost. If the bag has a mass of 36 kilograms, what is the minimum amount of work the gardener must do to lift the bag to a height of 0.5 meter?
A 36.5 J
B 72.0 J
C 176.4 J
D 352.8 J
Cluster 2

Use the information below and your knowledge of science to answer questions 40–42.

The following diagram represents a small section of DNA. In this diagram the letters T, C, A, and G stand for the nitrogen bases thymine, cytosine, adenine, and guanine. The codon chart below shows which mRNA codons code for which amino acids. For example, UUU = Phe means that the codon UUU codes for the amino acid abbreviated Phe. The letters U, C, A, and G stand for the nitrogen bases uracil, cytosine, adenine, and guanine.

mRNA Codon Chart

<table>
<thead>
<tr>
<th>First Base</th>
<th>Second Base</th>
<th>Third Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>UU</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>UUU</td>
<td>Phe</td>
<td>UAU</td>
</tr>
<tr>
<td>UUC</td>
<td>UCU</td>
<td>UAC</td>
</tr>
<tr>
<td>UUA</td>
<td>UCA</td>
<td>UAA</td>
</tr>
<tr>
<td>UUG</td>
<td>UCG</td>
<td>UAG</td>
</tr>
<tr>
<td>CU</td>
<td>Leu</td>
<td>CAU</td>
</tr>
<tr>
<td>CUU</td>
<td>CCA</td>
<td>CAC</td>
</tr>
<tr>
<td>CUC</td>
<td>CCC</td>
<td>CCA</td>
</tr>
<tr>
<td>CUA</td>
<td>CCG</td>
<td>CAG</td>
</tr>
<tr>
<td>CUG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AU</td>
<td>Iso</td>
<td>AAU</td>
</tr>
<tr>
<td>AUC</td>
<td>ACC</td>
<td>AAC</td>
</tr>
<tr>
<td>AUA</td>
<td>ACA</td>
<td>AAA</td>
</tr>
<tr>
<td>AUG</td>
<td>ACG</td>
<td>AAG</td>
</tr>
<tr>
<td>GA</td>
<td>Ala</td>
<td>GAA</td>
</tr>
<tr>
<td>GU</td>
<td>GC</td>
<td>GAG</td>
</tr>
<tr>
<td>GUU</td>
<td>GCU</td>
<td>GAC</td>
</tr>
<tr>
<td>GUC</td>
<td>GCC</td>
<td>GCA</td>
</tr>
<tr>
<td>GUA</td>
<td>GCG</td>
<td></td>
</tr>
<tr>
<td>GUG</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Question 40
Which amino acid chain could be translated from the Strand 1 section of DNA shown in the diagram?

A  Arg—Arg—Ala—Gly  
B  Met—Lys—Phe—Leu—Ala—Gly—Glu  
C  Thr—Lys—Asn—Arg—Pro—Asp  
D  Met—Phe—Lys—Asn—Arg—Pro—Iso

Question 41
A plant has a change in its DNA that makes it more resistant to a species of harmful bacteria. What will most likely happen as a result of this mutation?

A  The plant will not survive long enough to reproduce.  
B  The species of harmful bacteria will become extinct.  
C  Over time the number of resistant plants will increase.  
D  Over time the number of harmful bacteria will increase.

Question 42

<table>
<thead>
<tr>
<th>Amino Acid</th>
<th>0°C</th>
<th>25°C</th>
<th>50°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alanine</td>
<td>12.11</td>
<td>16.72</td>
<td>23.09</td>
</tr>
<tr>
<td>Aspartic acid</td>
<td>0.262</td>
<td>0.778</td>
<td>2.000</td>
</tr>
<tr>
<td>Isoleucine</td>
<td>1.826</td>
<td>2.229</td>
<td>3.034</td>
</tr>
<tr>
<td>Leucine</td>
<td>0.797</td>
<td>0.991</td>
<td>1.406</td>
</tr>
<tr>
<td>Phenylalanine</td>
<td>0.997</td>
<td>1.411</td>
<td>2.187</td>
</tr>
<tr>
<td>Serine</td>
<td>2.204</td>
<td>5.023</td>
<td>10.34</td>
</tr>
</tbody>
</table>

Based on the data in the table, which statement best describes the solubility of amino acids?

A  Amino acids become less soluble as solvent temperature increases.  
B  Aspartic acid is the least soluble amino acid at all solvent temperatures.  
C  Temperature has a greater effect on the solubility of serine than on any other amino acid.  
D  The solubility of amino acids increases as the temperature of the solvent increases.
SciencE

Fill a glass with water and drop in a thick rubber band. You’ll see that the rubber band sinks to the bottom of the glass. The mass of the rubber band is less than the mass of the water, so why doesn’t the rubber band float?

Density is a measure of an object’s mass per unit of volume. If an object is less dense than a liquid, it will float in the liquid. If it is denser than the liquid, it will sink. So a rubber band sinks because it has a greater density than water. In other words, 1 cubic centimeter (cm³) of rubber has a greater mass than 1 cm³ of water.

Even though you can’t make a rubber band float on water, you can still make it float. To do so, you need to make a density column.

**Liquid Rainbow**

Gather the materials in the following list:

- Small glass or clear plastic cup
- Thick rubber band
- Scissors
- Water
- Cooking oil
- Heavy syrup, such as pancake syrup or corn syrup (You can also use honey or molasses.)
- Small bowl
- Optional: food coloring, spoon, and ruler

Pour a small amount of water into a bowl and stir in a drop or two of food coloring. Slowly pour the water into the glass until the height of the water in the glass is about 2 centimeters.

Empty the bowl and pour in a small amount of cooking oil. Then slowly pour the oil into the glass until the total height of the liquid in the glass is about 4 centimeters.

Did the oil mix with the water, or did it stay in a separate layer? If the oil stayed in its own layer, did it sink or float on water? ____________
Empty the bowl again and pour in a small amount of syrup. Slowly pour the syrup into the glass until the total height of the liquid in the glass is about 6 centimeters.

Describe what happened when you added the syrup to the glass.

_____________________________________________________________

_____________________________________________________________

**Floating Layers**

Sketch your density column below. Label each of the layers.

Did the liquids mix together? Why or why not? ________________

_____________________________________________________________

What determines the order of the liquid layers in the glass? ______

_____________________________________________________________

List the liquids in order from least dense to most dense.__________

_____________________________________________________________


**Science Activity**

**Seeing Is Believing**

An object’s density can be calculated by dividing its mass by its volume.

\[
D = \frac{m}{v}
\]

You can use this formula to calculate the density of the liquids you used in your density column. Since you probably don’t have a graduated cylinder or a balance at home, you can’t accurately measure the mass and volume of the liquids yourself. Some sample data are listed in the table below. Use these data to calculate the density of the liquids.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Mass (g)</th>
<th>Volume (mL)</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>30.0</td>
<td>30.0</td>
<td></td>
</tr>
<tr>
<td>Cooking oil</td>
<td>27.6</td>
<td>30.0</td>
<td></td>
</tr>
<tr>
<td>Syrup</td>
<td>41.4</td>
<td>30.0</td>
<td></td>
</tr>
</tbody>
</table>

Based on your calculations, list the liquids in the table in order from least dense to most dense.

________________________________________

________________________________________

Does this list agree with what you observed in your density column? Explain. __________________________________________________________

________________________________________

**Hint:**

If a solid floats at the boundary between two liquids, its density is between that of the two liquids.

**Remember!**

The density of liquids is often given in grams per milliliter (g/mL), and the density of solids is often given in grams per cubic centimeter (g/cm³). A milliliter is the same as 1 cubic centimeter.

The density of liquids is often given in grams per milliliter (g/mL), and the density of solids is often given in grams per cubic centimeter (g/cm³). A milliliter is the same as 1 cubic centimeter.
Gather Some Solid Evidence

Use a pair of scissors to cut a piece of rubber band about 2 centimeters in length. Drop the piece into your density column.

What happens when you drop the rubber band into your density column? What does the position of the rubber band tell you about its density?

Try dropping some or all of the following solids into your density column:

- A penny
- A paper clip
- A small ice cube
- A plastic ring from the top of a soda or juice bottle
- Small pieces of butter, candle wax, chocolate, and charcoal
- A wooden toothpick
  (If necessary, break it in half so that it will fit in your glass.)
- Any other small objects you want to test

Use the diagram on page 329 to sketch the position of each object you added to your density column. Label the objects.

List the solids and liquids in your density column in order from least dense to most dense. Note: If two solids float at the same level, you will not be able to tell which one is denser than the other. In this case, list the two objects next to each other and circle them.

If you like, try adding other liquids to your density column. However, don’t add household cleaners such as bleach and ammonia to your density column. These cleaners could give off hazardous fumes if mixed together.

Be aware that if you add a liquid that is soluble in one of the liquids already in your column, the new liquid may not form its own layer. Instead, it may dissolve in one of the existing layers. Some other liquids to try include baby oil, liquid dish soap, rubbing alcohol, and mouthwash.
A Legendary Mystery

Archimedes was a mathematician and scientist who lived in ancient Greece. According to legend, Archimedes used his understanding of density to solve a crime. The king of Syracuse had received a new crown. The crown was supposed to be made of pure gold, but the king suspected that the gold had been mixed with silver.

Archimedes measured the density of the crown and found that it was less dense than pure gold. This evidence revealed that the king had been cheated by his crown maker. The gold in the crown had indeed been mixed with a lighter metal.

Suppose an archaeologist unearths a coin from the buried ruins of an ancient city. The coin has a mass of 4.18 grams and a volume of 0.23 cm³. Find the density of the coin. Show your work in the space below.

The density of gold is 19.3 g/cm³. Is the coin found by the archaeologist made of pure gold? Explain. ________________________________

Look at the sample results on page 343 to see how your answers compare.
**Science Answer Key**

**Objective 1**

**Question 1 (page 260)**

A Incorrect. Study the illustration again. Those plants growing in direct sunlight near the tree are larger than the smaller plants growing in the shade. If drier soil is a limiting factor, all the plants growing near the tree should be smaller.

B Incorrect. Study the illustration again. Those plants growing in direct sunlight near the tree are larger than the smaller plants growing in the shade. If the availability of minerals is a limiting factor, all the plants growing near the tree should be smaller.

C Correct. Only those plants growing in the shade are smaller. It is a reasonable hypothesis that the lower light level in the shaded area is limiting the growth of this plant species.

D Incorrect. Shaded soil will have a lower temperature than soil in direct sunlight. It is unlikely that slight temperature variations over a small distance would have a significant effect on plant growth.

**Question 2 (page 260)**

A Incorrect. On Day 12, some *P. caudatum* still exist in the culture medium.

B Incorrect. Given the slope of the graph, a small number of *P. caudatum* will still exist in the culture medium on Day 13.

C Correct. The population of *P. caudatum* should reach zero on Day 16 if the slope of the plotted line remains the same as between Days 6 and 12.

D Incorrect. According to the graph, the rate of decline of the population of *P. caudatum* is too great for them to survive to Day 18.

**Question 3 (page 261)**

A Incorrect. This graph indicates that the rate of growth is constant over time.

B Correct. This graph shows discontinuous intervals of rapid growth (after molting) followed by intervals of slow growth (after development of a new exoskeleton).

C Incorrect. This graph indicates that the rate of growth increases over time.

D Incorrect. This graph shows an initial period of growth followed by alternating periods of no growth (horizontal segments on the graph) and instantaneous growth (vertical segments). It takes time for any organism to grow, so the graph depicts an impossible relationship.

**Question 4 (page 261)**

A Incorrect. The graph shows that the rate of photosynthesis is significantly lower at 12°C than at 35°C.

B Incorrect. The graph shows that the rate of photosynthesis increases with increasing light intensity until it reaches a maximum value.

C Correct. The graph shows that after reaching a maximum value, the rate of photosynthesis begins to decrease as light intensity continues to increase.

D Incorrect. The rate of photosynthesis at 35°C is higher than the rate at 12°C for all light intensities shown on the graph.

**Question 5 (page 262)**

A, B, D Incorrect. These hypotheses can be tested with a well-designed experiment that uses a scientific approach.

C Correct. Opinions based on personal values, aesthetics, or other human preferences or prejudices cannot be tested by scientific methods.
Question 6 (page 262)
A Incorrect. A single serving of this fruit juice contains 29 grams of carbohydrates. A single serving provides only 10 percent of the total daily requirement for carbohydrates in a 2000-Calorie diet.

B Correct. The label states that one serving of this fruit juice provides 10 percent of the total daily requirement for carbohydrates in a 2000-Calorie diet:

\[ 29 \text{ g} \times 10 = 290 \text{ g} \] (100 percent)

C Incorrect. A single serving of this fruit juice contains 20 milligrams of sodium. Look for the phrase “Total Carbohydrate” on the nutrition label.

D Incorrect. A single serving of this fruit juice contains 350 milligrams of potassium. Look for the phrase “Total Carbohydrate” on the nutrition label.

Question 7 (page 263)
A, B, D Incorrect. These nonprescription medicines do not contain aspirin or other salicylate compounds.

C Correct. Bismuth subsalicylate is a salicylate compound. It is an active ingredient found in nausea medications. This compound should be avoided during recovery from a viral infection.

Question 8 (page 276)
A Incorrect. If a patient were given pure water intravenously, the concentration of water outside cells would increase relative to the concentration of water inside cells. Therefore, water would tend to move into cells and cause cells to swell rather than shrivel.

B Incorrect. Osmosis refers only to the movement of water, not to the movement of salt or other dissolved substances.

C Incorrect. If a patient were given pure water, the concentration of dissolved substances in the fluid outside cells would decrease. This would cause water to move into cells and cause the cells to swell and burst.

D Correct. The fluid inside and outside cells is not pure water; it contains dissolved substances such as salts and sugars. To prevent cells from shriveling or swelling because of osmosis, it is best to give a patient an intravenous solution with a concentration of dissolved substances that is close to the concentration of dissolved substances in the body’s fluids.

Question 9 (page 276)
A Correct. As the athlete’s heart rate increases, the rate at which blood travels through her body also increases. Blood carries oxygen from the lungs to body cells. Therefore, as blood travels faster through the body, more oxygen can be transported to body cells.

B Incorrect. Insulin is a hormone that promotes a reduction of glucose in the blood. Insulin does not affect the level of oxygen in the blood.

C Incorrect. If the athlete’s breathing were to become more shallow, her body would take in less oxygen with each breath.

D Incorrect. The athlete’s sweat glands are likely to become more active as she runs. Perspiration helps maintain body temperature during exercise. However, increased perspiration will not help muscle cells meet their increased oxygen demand.

Question 10 (page 276)
A Correct. All of the offspring of this cross will have the phenotype for high yields and rapid maturation (all hhMm). Only the genotypes for yield and maturation rate are needed to answer this question. The genotypes for plant height and kernel color can be ignored.

B Incorrect. A cross of these genotypes will produce a generation in which all the plants will exhibit the phenotype for rapid maturation (all Mm), but none will exhibit the phenotype for high yields (all Hh).

C Incorrect. A cross of these genotypes will produce a generation in which one-fourth of the plants will have the phenotype for high yields and rapid maturation (one-fourth hhMm).

D Incorrect. A cross of these genotypes will produce a generation in which all the plants will exhibit the phenotype for low yields (all Hh), and only half will exhibit the phenotype for rapid maturation (half Mm and half mm).
**Question 11 (page 277)**

A **Correct.** In some cases, more than one codon can code for the same amino acid. It is possible for a mutation to occur in DNA and for the new codon to still code for the same amino acid. In such a case, the protein that results from the mutated DNA will be identical to the protein that results from normal DNA.

B **Incorrect.** If the codon GAG were a stop signal, only part of the hemoglobin protein would be made. This partial protein would probably not be able to function normally.

C **Incorrect.** There are no codons that are unreadable or skipped over during protein synthesis.

D **Incorrect.** Each tRNA corresponds to only one codon; it can carry only one type of amino acid.

**Question 12 (page 278)**

A **Incorrect.** Histamine is a polar molecule. Because polar molecules are not soluble in lipids, they cannot easily diffuse across the cell membrane. Furthermore, an increase in histamine concentration would likely lead to a worsening of allergy symptoms.

B **Correct.** An antihistamine, which is chemically similar to histamine, can bind to these receptors without causing the reactions within the cell that lead to the inflammatory response. Histamine, in turn, cannot bind to the receptors that are blocked by antihistamines.

C **Incorrect.** If the number of histamine receptors on a cell's surface were to increase, there would be more opportunities for histamine to bind to these receptors and trigger an allergic reaction. Therefore, an increase in the production of histamine receptors would likely lead to a worsening of allergy symptoms.

D **Incorrect.** Histamine is a polar molecule. Because polar molecules are not soluble in lipids, they cannot easily pass through the cell membrane.

**Question 13 (page 278)**

A **Incorrect.** If Duchenne muscular dystrophy were an autosomal recessive disorder, then anyone who carried at least one copy of the mutated gene would have the disorder. However, two of the females in the second generation and one female in the fourth generation of the pedigree are carriers of the mutated gene but do not have the disorder. So the disorder is not autosomal recessive.

B **Incorrect.** If Duchenne muscular dystrophy were an autosomal dominant disorder, then anyone who carried at least one copy of the mutated gene would have the disorder. However, two of the females in the second generation and one female in the fourth generation of the pedigree are carriers of the mutated gene but do not have the disorder. So the disorder is not autosomal dominant.

C **Correct.** The gene is located on the X chromosome. In the pedigree below, X^D^ represents an X chromosome with a copy of the normal, dominant gene, and X^d^ represents an X chromosome with a copy of the mutated, recessive gene. Females with one copy of the mutated allele are not affected by the disorder, because they also have an X chromosome with a normal, dominant allele. Males with one copy of the mutated allele, however, will have the disorder because they have only one X chromosome.

D **Incorrect.** If Duchenne muscular dystrophy were a sex-linked dominant disorder, a daughter would have the disorder if she inherited one copy of the mutated allele. However, the daughters of the first generation in the pedigree have one copy of the mutated allele but do not have the disorder. The disorder is not sex-linked dominant.
Question 14 (page 279)
A Incorrect. Each F1 parent has a genotype of Bb. Because both F1 parents are heterozygous, not all the F2 mice will have BB genotypes, nor will all the F2 mice be black. See the Punnett square in the explanation for the correct answer.
B Incorrect. Each F1 parent has a genotype of Bb. Because both F1 parents are heterozygous, not all the F2 mice will have bb genotypes, nor will all the F2 mice be white. See the Punnett square in the explanation for the correct answer.
C Correct. The F1 generation has a genotype of Bb. Each mouse inherits a B allele from its BB parent and a b allele from its bb parent. The B allele is dominant because a mouse with a genotype of Bb has a black coat. The F2 generation is the result of a heterozygous cross (Bb × Bb). The expected genotypes are 25% BB, 50% Bb, and 25% bb. Because both BB and Bb mice have black coats, the expected phenotypes are 75% black coats and 25% white coats.
D Incorrect. Each F1 parent has a genotype of Bb. It is possible for an F2 mouse to inherit a B allele from one parent and a b allele from the other. These mice would NOT have BB or bb genotypes. In addition, the Punnett square in the explanation for the correct answer shows that there will be more black F2 mice than white F2 mice.

Question 15 (page 286)
A Correct. The two-tone color of the shell is most likely a form of camouflage that makes detection by predatory birds and fish difficult. To get this question correct, it is not important to know everything (or anything) about common purple snails. But it is very important to read the question carefully and understand the basic concept of natural selection.
B Incorrect. The common purple snail is unable to swim and relies on random encounters with prey organisms. Shell color plays no role in feeding or in competition with other organisms.
C Incorrect. Shell color is not an adaptation to average water temperature.
D Incorrect. Shell color offers no selective advantage for locating prey or increasing the number of encounters with prey organisms.

Question 16 (page 286)
A Incorrect. There is no evidence that the bacteria in yogurt prevent or stop viral reproduction.
B Incorrect. The bacteria in yogurt do not eat other bacteria.
C Correct. Antibiotics cannot tell the difference between helpful and harmful bacteria. This means that most antibiotics will act upon many types of bacteria in the human body, including bacteria in the digestive tract needed for good health.
D Incorrect. Dietary fiber is the part of a plant’s tissue that cannot be digested by humans. Yogurt is made from milk, not plants, so it does not contain dietary fiber.

Question 17 (page 287)
A Incorrect. Jawed fishes descended from primitive jawless fishes. The diagram does not show the evolutionary history of jawless fishes. Therefore, it is not possible to know whether they are extinct.
B Incorrect. According to the diagram, amphibians evolved from lobe-finned fishes but not directly from lungfish.
C Correct. All ray-finned and lobe-finned fishes have bony skeletons. The diagram shows that ray-finned and lobe-finned fishes evolved from an older type of bony fish.
D Incorrect. According to the diagram, sturgeon and coelacanths both evolved from a common ancestor, the bony fishes. Sharks evolved from cartilaginous fishes.

Question 18 (page 288)

A Correct. When this evergreen cone opens, it releases small seeds with papery “wings.” The lightweight seeds are carried on wind currents and are deposited some distance from the parent plant.

B Incorrect. These berries may be eaten by a bird or mammal. The seeds in the berries will pass unharmed through the animal’s digestive tract. They are then deposited at a location some distance from the parent plant.

C Incorrect. Seeds such as this cocklebur catch in the fur of animals or on clothing and are carried some distance from the parent plant.

D Incorrect. Coconuts may germinate on the beach close to their parent trees, or ocean currents may carry them to a different location.

Question 19 (page 288)

A Incorrect. Thick soil forms in tropical rain forests because warmth and moisture cause intense chemical weathering and the rapid decay of organic matter. The shallow roots of rain-forest plants have little effect on these processes.

B Incorrect. The vegetation in a tropical rain forest does limit erosion. However, the lack of significant erosion does not cause nutrients to build up in the soil. Rather, nutrients are quickly removed by plants.

C Correct. Organic matter on the forest floor is rapidly decomposed and recycled because of the warm, moist conditions and the presence of numerous decomposers. Nutrients are quickly removed by plants, so most nutrients are stored in the biomass of the organisms that live in the rain forest.

D Incorrect. The vegetation in a tropical rain forest may moderate daily temperature variations, but the high temperatures throughout the year are due to the location near the equator.

Question 20 (page 289)

A Correct. A secondary consumer feeds on primary consumers. A tertiary consumer feeds on secondary consumers. Notice how the hawk and the snake fill these different roles in these food chains.

Wood rat (primary consumer) → snake (secondary consumer)
Termite (primary consumer) → lizard (secondary consumer) → snake (tertiary consumer)
Wood rat (primary consumer) → hawk (secondary consumer)
Termite (primary consumer) → lizard (secondary consumer) → hawk (tertiary consumer)

B Incorrect. The lizard feeds on primary consumers, so it is a secondary consumer in this food web. The wood rat does not feed on any primary consumers, so it is not a secondary consumer.

C Incorrect. The hawk is both a secondary and a tertiary consumer in this food web. However, the termite does not feed on any primary consumers, so it is not a secondary consumer.

D Incorrect. The snake is both a secondary and a tertiary consumer in this food web. According to the food web, the lizard feeds on two primary consumers, the moth and the termite, but no secondary consumers.

Question 21 (page 289)

A Incorrect. Although mosquitoes can transmit diseases such as malaria and yellow fever, they play no role in the transmission of tuberculosis. The lungs are the usual site of such an infection.

B Incorrect. It may be possible to spread tuberculosis bacteria through a blood transfusion, but it is unlikely. These bacteria most often reside in the lungs, and blood donors are carefully screened.

C Correct. Tuberculosis is spread from person to person through the air. The lungs are the usual site of this disease. When a person with active tuberculosis coughs, sneezes, or speaks, droplets containing the bacteria are expelled from the lungs. If another person inhales these airborne droplets, that person can become infected with tuberculosis. To get this question correct, it is not important to know everything (or anything) about tuberculosis. But it is very important to read the question carefully and understand that diseases of the lungs will most likely be transmitted through the air.

D Incorrect. The bacteria that cause tuberculosis are not transmitted by contaminated water. They reside only in living hosts. The lungs are the usual site of a tuberculosis infection.
Objective 4

**Question 22 (page 303)**

Find the formula for density in the formula chart on page 248:

\[
\text{Density} = \frac{\text{mass}}{\text{volume}}
\]

The mass of the liquid is equal to the reading on the scale minus the mass of the empty cylinder:

\[
201.76\, \text{g} - 87.76\, \text{g} = 114\, \text{g}
\]

The volume of liquid can be determined from the picture of the graduated cylinder. The volume is 45 mL. Now use the formula:

\[
\text{Density of the liquid} = \frac{114\, \text{g}}{45\, \text{mL}} = 2.5\, \text{g/mL}
\]

If you add a zero after the 5 or before the 2, the answer would still be correct.

**Question 23 (page 303)**

A Incorrect. The density of steel is greater than the density of seawater. However, this fact does not explain why a ship with a steel hull is able to float.

B Incorrect. The buoyant force on the ship is equal to the weight of the seawater displaced by the ship. If the weight of seawater displaced by the ship were less than the weight of the ship itself, the ship would sink, not float.

C Correct. The ship has a hollow steel hull, which means that most of the volume of the ship is taken up by air. Because air is much less dense than seawater, the density of the ship as a whole (hull plus air) is less than the density of seawater. Therefore, the volume of seawater that the ship displaces weighs more than the ship, and the ship floats.

D Incorrect. The buoyant force on an object is always equal to the weight of the liquid displaced by the object. Whether the object sinks or floats depends on the weight of the object compared to the weight of the liquid it displaces.

**Question 24 (page 303)**

A Incorrect. Sodium bicarbonate (NaHCO₃) has a lower molecular mass than sodium carbonate (Na₂CO₃) does. Even if sodium bicarbonate did have a greater mass, that would not account for the difference in mass between the reactant and the solid product.

B Correct. According to the law of conservation of mass, the mass of the reactants in a chemical reaction is equal to the mass of the products. Because the mass of the product sodium carbonate does not equal the mass of the reactant sodium bicarbonate, there must be other products. These products are most likely gases because sodium carbonate is the only solid product of the reaction. The chemical reaction is:

\[
2\text{NaHCO}_3(s) \rightarrow \text{Na}_2\text{CO}_3(s) + \text{CO}_2(g) + \text{H}_2\text{O}(g)
\]

The mass of sodium carbonate plus the mass of the carbon dioxide plus the mass of the water is equal to the mass of sodium bicarbonate.

C Incorrect. The law of conservation of mass states that matter is not destroyed during a chemical reaction. All the atoms that are present in the reactants of a chemical reaction will also appear in the products.

D Incorrect. The law of conservation of mass states that the total mass of the reactants in a chemical reaction is equal to the total mass of the products.
Question 25 (page 304)

B Correct. The graph shows that the solubility of KNO₃ decreases as the temperature decreases. At 60°C a saturated solution can hold about 95 grams of KNO₃ per 100 grams of water. At 25°C a saturated solution can hold only about 30 grams of KNO₃ per 100 grams of water. To find the mass of KNO₃ that settles out of the solution, find the difference between the solubilities:

\[ 95 \text{ g} - 30 \text{ g} = 65 \text{ g} \]

So about 65 grams of KNO₃ will have settled out of the solution once it has cooled.

Question 26 (page 304)

A Correct. The sugar added to this beaker is composed of small granules, so it has a large total surface area. The greater the surface area of the particles, the faster they will dissolve. In addition, the solution is being stirred with a stirring rod. Stirring brings more water in contact with the surface of the sugar, which increases the dissolving rate. Finally, the water in this beaker is hot. For solid solutes such as sugar, increasing the temperature increases the rate at which the solute dissolves.

B Incorrect. The water in this beaker is below room temperature. Because of the lower temperature, the sugar added to this beaker will tend to dissolve more slowly than sugar added to warmer water. In addition, the sugar is in large cubes rather than small granules, which decreases the surface area. The sugar added to this beaker will tend to dissolve more slowly than sugar with a smaller particle size.

C Incorrect. Potassium chloride (KCl) is a solid. In general, the solubility of solids in water increases with temperature. Therefore, KCl and other potassium fertilizers will become more soluble in the lake as the amount of thermal pollution increases.

D Incorrect. Quartz crystals are solids. In general, the solubility of solids in water increases with temperature. However, most minerals have a low solubility in water. For instance, quartz has a solubility of less than 10 parts per million. As a lake’s temperature rises, the increase in the solubility of minerals would not significantly increase and harm fish.

Question 27 (page 305)

A Correct. The solubility of a gas in water decreases as the temperature increases. As thermal pollution causes the temperature of the lake to rise, the amount of oxygen dissolved in the lake will decrease. Fish need dissolved oxygen in order to survive, so they may be negatively affected by such a change. To get this question correct, it is important to read the question carefully and to understand how temperature affects the solubility of a gas.

B Incorrect. The solubility of a gas in water decreases as the temperature increases. As the temperature of the lake rises, the solubility of carbon dioxide gas will decrease.

C Incorrect. Potassium chloride (KCl) is a solid. In general, the solubility of solids in water increases with temperature. Therefore, KCl and other potassium fertilizers will become more soluble in the lake as the amount of thermal pollution increases.

D Incorrect. Quartz crystals are solids. In general, the solubility of solids in water increases with temperature. However, most minerals have a low solubility in water. For instance, quartz has a solubility of less than 10 parts per million. As a lake’s temperature rises, the increase in the solubility of minerals would not significantly increase and harm fish.

Question 28 (page 305)

D Correct. According to the law of conservation of mass, each side of a balanced equation must have the same number of atoms of each element. So the balanced equation is:

\[ 2\text{Al}(s) + 3\text{CuSO}_4(aq) \rightarrow \text{Al}_2(\text{SO}_4)_3(aq) + 3\text{Cu}(s) \]

Each side of the equation has 2 aluminum atoms (Al), 3 copper atoms (Cu), 3 sulfur atoms (S), and 12 oxygen atoms (O).
Question 29 (page 321)

C Correct. When the epicenter is 4,000 kilometers from the seismograph, it takes 7 minutes for the P wave to arrive and 12 minutes 30 seconds for the S wave to arrive. The S wave arrives 5 minutes 30 seconds after the P wave:

\[12 \text{ min } 30 \text{ s} - 7 \text{ min} = 5 \text{ min } 30 \text{ s}\]

If the P wave arrives at 05:26:00, then the S wave arrives at 05:31:30 because:

\[05:26:00 + 00:05:30 = 05:31:30\]

Question 30 (page 321)

B Correct. From the information given, we know:

- time = 4 s
- Force = 1500 N
- distance = 1.5 m

We need to calculate the power.

The two formulas we need to use are:

\[P = \frac{W}{t}\] and \[W = Fd\]

Substitute these values into the work formula:

\[Work = 1500 \text{ N} \times 1.5 \text{ m}\]
\[1500 \text{ N} \times 1.5 \text{ m} = 2250 \text{ Nm}\]
\[2250 \text{ Nm} = 2250 \text{ J}\]

Now use the power formula:

\[Power = \frac{W}{t}\]
\[\frac{2250 \text{ J}}{4.0 \text{ s}} = 562.5 \text{ J/s}\]
\[562.5 \text{ J/s} = 562.5 \text{ W}\]
\[562.5 \text{ W} = 563 \text{ W}\]

So, the weight lifter uses about 563 watts of power.

Question 31 (page 322)

D Correct. Because the four-door family car has the greatest mass, it would strike the wall with the greatest force.

Question 32 (page 322)

A Incorrect. Although Bulbs 2 and 3 are wired in parallel, their branches are wired in series with Bulb 1. If electric current cannot pass through Bulb 1, the circuit will be broken and none of the bulbs will remain lit.

B Incorrect. In this circuit the three bulbs are wired in series. As a result, the circuit will be broken and none of the bulbs will remain lit if Bulb 1 burns out.

C Correct. In this series Bulbs 1 and 3 are wired in parallel, and their branches are wired in series with Bulb 2. If Bulb 1 burns out, there will still be a complete circuit that includes the battery, Bulb 2, and Bulb 3. Therefore, Bulbs 2 and 3 will remain lit even though Bulb 1 burns out.

D Incorrect. Although Bulbs 1 and 3 are wired in parallel with Bulb 2, they are wired in series with each other. As a result, electric current will not pass through Bulb 3 if it cannot pass through Bulb 1. Bulb 2, however, will remain lit even if Bulb 1 burns out; there will still be a complete circuit that includes the battery and Bulb 2.

Question 33 (page 323)

A Incorrect. The amplitude of the waves would increase only if the waves were carrying more energy. The energy of the waves is determined by the source—the siren. Because the source does not change, the amplitude of the waves remains constant.

B Incorrect. The wavelength of the waves decreases in front of the moving ambulance.

C Incorrect. The speed of a sound wave depends only on the medium it travels through, not on the motion of the source. The speed of these sound waves remains constant.

D Correct. As the ambulance moves toward the man, each sound wave is produced at a point closer to him. As a result, the wave crests in front of the moving ambulance are closer together than if the ambulance were not moving. Therefore, the wavelength of the waves is shorter in front of the moving ambulance. Because the wave crests are closer together, more of them reach the man in a given unit of time. Therefore, the frequency increases, and the sound that he hears is a higher pitch.
Question 34 (page 323)

D Correct. To solve this problem, use the formula for acceleration in the formula chart on page 248:

\[ a = \frac{v_f - v_i}{\Delta t} \]

Since the stone is dropped, the acceleration of the stone due to gravity is 9.8 m/s². This constant is listed in the constants/conversions chart. The initial velocity of the stone is 0 m/s. The change in time is 2.30 seconds. Substitute these values into the formula and solve for the final velocity of the stone.

\[ 9.8 \text{ m/s}^2 = \frac{v_f - 0 \text{ m/s}}{2.30 \text{ s}} \]

\[ v_f = 22.54 \text{ m/s} \]

The velocity of the stone when it hits the river is 22.54 m/s ≈ 22.5 m/s.

Question 35 (page 323)

A Incorrect. Even though the crate is moved farther than the other objects, its movement does not involve the greatest amount of work. The work done to move the crate is calculated by the formula: Work = force × distance or 10 N × 9.8 m = 98 J.

B Correct. Look at the formula on chart page 10 to find the formula for work:

\[ \text{Work} = \text{force} \times \text{distance}. \]

The force used to move the wagon is 50 newtons, and the wagon is moved a distance of 5.2 meters. Substitute these values into the formula.

\[ \text{Work} = 50 \text{ N} \times 5.2 \text{ m} = 260 \text{ J}. \]

C Incorrect. Even though a greater force is used to move the sled than either the crate or the wagon, its movement does not involve the greatest amount of work. The work done to move the sled is calculated by the formula:

\[ \text{Work} = \text{force} \times \text{distance} \]

\[ 90 \text{ N} \times 2.3 \text{ m} = 207 \text{ J}. \]

D Incorrect. Work is accomplished only when the force applied to an object results in the movement of the object. Because the car does not move, no work is done: 150 N × 0 m = 0 J.

Question 36 (page 325)

A Incorrect. According to the food web, beetles feed on snails, slugs, and springtails, and springtails feed on dead organic matter, bacteria, fungi, and nematodes. There is no indication that spiders eat the same food as beetles and springtails.

B Incorrect. None of the information given indicates that the space available has changed.

C Correct. When the spider population increased after the removal of the centipedes, there were more spiders to prey on beetles and springtails. As a result, the populations of these insects decreased.

D Incorrect. Herbivores feed on plants. The beetles and springtails in this food web do not feed on plants (although one food source for springtails is dead plant material). Therefore, the beetles and springtails in the compost pile are unlikely to compete with herbivores for food.

Question 37 (page 325)

A Incorrect. To answer this question, the students would need to make measurements of the moisture content of both compost piles at regular time intervals.

B Incorrect. To answer this question, the students would need to use different types of organic material in the two compost piles.

C Correct. The variable that the students change is the amount of water the compost piles receive. They can determine in which pile more organisms live (the moist one or the dry one) by random sampling of the number of different types of organisms in each pile.

D Incorrect. To answer this question, the students would need to either add carnivores to one of the piles or remove carnivores from one of the piles.

Question 38 (page 325)

A Incorrect. The cells of fungi do not contain chloroplasts. Fungi do not make their own food from sunlight. Instead, fungi obtain food by absorbing it into their cells. Many fungi feed on the remains of dead organisms.

B Incorrect. The cells of bacteria lack most types of organelles, including chloroplasts.
Then the mRNA strand is translated to produce an amino acid chain. Use the codon chart to determine which amino acid matches each mRNA codon.

\[
\text{mRNA} \quad \text{AUG} - \text{AAG} - \text{UUU} - \text{UUG} - \text{GCU} - \text{GGC} - \text{GAG}
\]

\[
\begin{align*}
\text{Amino acid} & \quad \text{Translation} \\
\text{start} & \quad \text{Met} - \text{Lys} - \text{Phe} - \text{Leu} - \text{Ala} - \text{Gly} - \text{Glu}
\end{align*}
\]

The amino acid chain that results is Met—Lys—Phe—Leu—Ala—Gly—Glu

**Question 41 (page 327)**

A Incorrect. This mutation is beneficial to the plant. Therefore, the mutation makes it more likely that the plant will live long enough to pass on its genes to offspring.

B Incorrect. There is no evidence to suggest that the species of bacteria needs the plant species in order to survive. Even if resistance to the bacteria spreads throughout the plant species, there may be other types of organisms that the bacteria could use as hosts or food sources.

C Correct. This mutation is beneficial to the plant. Therefore, this change is likely to be passed on to offspring. Because the offspring would be resistant to the harmful bacteria, they would be more likely to survive and pass on the mutation to a third generation. Eventually the number of resistant plants could be quite large.

D Incorrect. This mutation makes the plant resistant to the harmful bacteria. If this mutation is passed on through many generations of plants, it will be more likely to have a negative effect on the bacteria than a positive effect.

**Question 42 (page 327)**

A Incorrect. The amino acids listed in the table become more soluble as the temperature increases.

B Incorrect. At 50°C, leucine is less soluble in water than aspartic acid is.

C Incorrect. As the temperature increases from 0°C to 50°C, the solubility of aspartic acid increases more than the solubility of serine. So, temperature has a greater effect on the solubility of aspartic acid than on the solubility of serine.

D Correct. Each of the amino acids listed in the table becomes more soluble in water as the temperature of the water increases from 0°C to 25°C to 50°C.
**Science Activity**

**Liquid Rainbow (page 328)**

Did the oil mix with the water, or did it stay in a separate layer? If the oil stayed in its own layer, did it sink or float on water? The oil stayed in a separate layer that floated on top of the water. Describe what happened when you added the syrup to the glass. The syrup stayed in its own layer and sank to the bottom of the glass.

**Floating Layers (page 329)**

The sketch of your density column will depend on the types of objects you add to it. A sample sketch is shown below.

Did the liquids mix together? Why or why not? No. The liquids didn’t mix together, because they are not very soluble in one another at room temperature and because they have different densities.

What determines the order of the liquid layers in the glass? The density of the liquids determines their order. The densest liquid is on the bottom of the glass, and the least-dense liquid is on the top.

List the liquids in order from least dense to most dense. The correct order is: cooking oil, water, syrup.

**Seeing Is Believing (page 330)**

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Mass (g)</th>
<th>Volume (mL)</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>30.0</td>
<td>30.0</td>
<td>1.00 g/mL</td>
</tr>
<tr>
<td>Cooking oil</td>
<td>27.6</td>
<td>30.0</td>
<td>0.92 g/mL</td>
</tr>
<tr>
<td>Syrup</td>
<td>41.4</td>
<td>30.0</td>
<td>1.38 g/mL</td>
</tr>
</tbody>
</table>

Based on your calculations, list the liquids in the table in order from least dense to most dense. The correct order is: cooking oil, water, syrup.

Does this list agree with what you observed in your density column? Explain. Yes. My calculations and my density column both show that cooking oil is less dense than water and that syrup is denser than water.

**Gather Some Solid Evidence (page 331)**

What happens when you drop the rubber band into your density column? What does the position of the rubber band tell you about its density? The rubber band floats at the boundary between the syrup and the water. The rubber band is denser than water but less dense than syrup.

List the solids and liquids in your density column in order from least dense to most dense. Your answer will depend on the solids you used. Sample: toothpick, cooking oil, plastic ring, water, chocolate, rubber band, syrup, paper clip, penny

**A Legendary Mystery (page 332)**

\[
Density = \frac{mass}{volume}
\]

\[
D = \frac{m}{v}
\]

\[
D = \frac{4.18 \text{ g}}{0.23 \text{ cm}^3} = 18.2 \text{ g/cm}^3
\]

The coin has a density of 18.2 g/cm³.

Is the coin found by the archaeologist made of pure gold? Explain. No. The density of the coin (18.2 g/cm³) is less than the density of pure gold (19.3 g/cm³).
### Grades 9, 10, and 11 Exit Level Mathematics Chart

#### LENGTH

<table>
<thead>
<tr>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilometer = 1000 meters</td>
<td>1 mile = 1760 yards</td>
</tr>
<tr>
<td>1 meter = 100 centimeters</td>
<td>1 mile = 5280 feet</td>
</tr>
<tr>
<td>1 centimeter = 10 millimeters</td>
<td>1 yard = 3 feet</td>
</tr>
<tr>
<td></td>
<td>1 foot = 12 inches</td>
</tr>
</tbody>
</table>

#### CAPACITY AND VOLUME

<table>
<thead>
<tr>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 liter = 1000 milliliters</td>
<td>1 gallon = 4 quarts</td>
</tr>
<tr>
<td></td>
<td>1 gallon = 128 ounces</td>
</tr>
<tr>
<td></td>
<td>1 quart = 2 pints</td>
</tr>
<tr>
<td></td>
<td>1 pint = 2 cups</td>
</tr>
<tr>
<td></td>
<td>1 cup = 8 ounces</td>
</tr>
</tbody>
</table>

#### MASS AND WEIGHT

<table>
<thead>
<tr>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilogram = 1000 grams</td>
<td>1 ton = 2000 pounds</td>
</tr>
<tr>
<td>1 gram = 1000 milligrams</td>
<td>1 pound = 16 ounces</td>
</tr>
</tbody>
</table>

#### TIME

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year = 365 days</td>
<td></td>
</tr>
<tr>
<td>1 year = 12 months</td>
<td></td>
</tr>
<tr>
<td>1 year = 52 weeks</td>
<td></td>
</tr>
<tr>
<td>1 week = 7 days</td>
<td></td>
</tr>
<tr>
<td>1 day = 24 hours</td>
<td></td>
</tr>
<tr>
<td>1 hour = 60 minutes</td>
<td></td>
</tr>
<tr>
<td>1 minute = 60 seconds</td>
<td></td>
</tr>
</tbody>
</table>

Continued on the next side
# Grades 9, 10, and 11 Exit Level Mathematics Chart

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>rectangle</th>
<th>( P = 2l + 2w ) or ( P = 2(l + w) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference</td>
<td>circle</td>
<td>( C = 2\pi r ) or ( C = \pi d )</td>
</tr>
<tr>
<td>Area</td>
<td>rectangle</td>
<td>( A = lw ) or ( A = bh )</td>
</tr>
<tr>
<td></td>
<td>triangle</td>
<td>( A = \frac{1}{2} bh ) or ( A = \frac{bh}{2} )</td>
</tr>
<tr>
<td></td>
<td>trapezoid</td>
<td>( A = \frac{1}{2} (b_1 + b_2)h ) or ( A = \frac{(b_1 + b_2)h}{2} )</td>
</tr>
<tr>
<td></td>
<td>circle</td>
<td>( A = \pi r^2 )</td>
</tr>
<tr>
<td>Surface Area</td>
<td>cube</td>
<td>( S = 6s^2 )</td>
</tr>
<tr>
<td></td>
<td>cylinder (lateral)</td>
<td>( S = 2\pi rh )</td>
</tr>
<tr>
<td></td>
<td>cylinder (total)</td>
<td>( S = 2\pi rh + 2\pi r^2 ) or ( S = 2\pi r(h + r) )</td>
</tr>
<tr>
<td></td>
<td>cone (lateral)</td>
<td>( S = \pi rl )</td>
</tr>
<tr>
<td></td>
<td>cone (total)</td>
<td>( S = \pi rl + \pi r^2 ) or ( S = \pi r(l + r) )</td>
</tr>
<tr>
<td></td>
<td>sphere</td>
<td>( S = 4\pi r^2 )</td>
</tr>
<tr>
<td>Volume</td>
<td>prism or cylinder</td>
<td>( V = Bh^* )</td>
</tr>
<tr>
<td></td>
<td>pyramid or cone</td>
<td>( V = \frac{1}{3} Bh^* )</td>
</tr>
<tr>
<td></td>
<td>sphere</td>
<td>( V = \frac{4}{3} \pi r^3 )</td>
</tr>
<tr>
<td>*( B ) represents the area of the Base of a solid figure.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## Pi
\[ \pi \approx 3.14 \quad \text{or} \quad \pi \approx \frac{22}{7} \]

## Pythagorean Theorem
\[ a^2 + b^2 = c^2 \]

## Distance Formula
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

## Slope of a Line
\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

## Midpoint Formula
\[ M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

## Quadratic Formula
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

## Slope-Intercept Form of an Equation
\[ y = mx + b \]

## Point-Slope Form of an Equation
\[ y - y_1 = m(x - x_1) \]

## Standard Form of an Equation
\[ Ax + By = C \]

## Simple Interest Formula
\[ I = prt \]
### Formula Chart for Grades 10–11 Science Assessment

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D = \frac{m}{v} )</td>
<td>Density = ( \frac{\text{mass}}{\text{volume}} )</td>
</tr>
<tr>
<td>( Q = (m)(\Delta T)(C_p) )</td>
<td>Heat gained or lost by water = (mass in grams)(change in temperature)(specific heat)</td>
</tr>
<tr>
<td>( s = \frac{d}{t} )</td>
<td>Speed = ( \frac{\text{distance}}{\text{time}} )</td>
</tr>
<tr>
<td>( a = \frac{v_f - v_i}{\Delta t} )</td>
<td>Acceleration = ( \frac{\text{final velocity} - \text{initial velocity}}{\text{change in time}} )</td>
</tr>
<tr>
<td>( p = mv )</td>
<td>Momentum = mass ( \times ) velocity</td>
</tr>
<tr>
<td>( F = ma )</td>
<td>Force = mass ( \times ) acceleration</td>
</tr>
<tr>
<td>( W = Fd )</td>
<td>Work = force ( \times ) distance</td>
</tr>
<tr>
<td>( P = \frac{W}{t} )</td>
<td>Power = ( \frac{\text{work}}{\text{time}} )</td>
</tr>
<tr>
<td>( % \text{ efficiency} = \frac{\text{work output}}{\text{work input}} \times 100 )</td>
<td>% efficiency = ( \frac{W_o}{W_i} \times 100 )</td>
</tr>
<tr>
<td>( KE = \frac{1}{2} (mv^2) )</td>
<td>Kinetic energy = ( \frac{1}{2} (\text{mass} \times \text{velocity}^2) )</td>
</tr>
<tr>
<td>( GPE = mgh )</td>
<td>Gravitational potential energy = mass ( \times ) acceleration due to gravity ( \times ) height</td>
</tr>
<tr>
<td>( E = mc^2 )</td>
<td>Energy = mass ( \times ) (speed of light)(^2)</td>
</tr>
<tr>
<td>( v = f\lambda )</td>
<td>Velocity of a wave = frequency ( \times ) wavelength</td>
</tr>
<tr>
<td>( I = \frac{V}{R} )</td>
<td>Current = ( \frac{\text{voltage}}{\text{resistance}} )</td>
</tr>
<tr>
<td>( P = VI )</td>
<td>Electrical power = voltage ( \times ) current</td>
</tr>
<tr>
<td>( E = Pt )</td>
<td>Electrical energy = power ( \times ) time</td>
</tr>
</tbody>
</table>

### Constants/Conversions

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g )</td>
<td>acceleration due to gravity = 9.8 m/s(^2)</td>
</tr>
<tr>
<td>( c )</td>
<td>speed of light = ( 3 \times 10^8 ) m/s</td>
</tr>
<tr>
<td>speed of sound</td>
<td>343 m/s at 20°C</td>
</tr>
<tr>
<td>1 cm(^3)</td>
<td>1 mL</td>
</tr>
<tr>
<td>1 wave/second</td>
<td>1 hertz (Hz)</td>
</tr>
<tr>
<td>1 calorie (cal)</td>
<td>4.18 joules</td>
</tr>
<tr>
<td>1000 calories (cal)</td>
<td>1 Calorie (Cal) = 1 kilocalorie (kcal)</td>
</tr>
<tr>
<td>newton (N)</td>
<td>kg m/s(^2)</td>
</tr>
<tr>
<td>joule (J)</td>
<td>Nm</td>
</tr>
<tr>
<td>watt (W)</td>
<td>J/s = Nm/s</td>
</tr>
<tr>
<td>volt (V)</td>
<td>ampere (A)</td>
</tr>
<tr>
<td>ohm (Ω)</td>
<td></td>
</tr>
</tbody>
</table>
### Periodic Table of the Elements

<table>
<thead>
<tr>
<th>Group</th>
<th>Period</th>
<th>Block</th>
<th>Atomic Number</th>
<th>Symbol</th>
<th>Element</th>
<th>Atomic Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>s</td>
<td>1</td>
<td>H</td>
<td>Hydrogen</td>
<td>1.008</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>s</td>
<td>2</td>
<td>He</td>
<td>Helium</td>
<td>4.0026</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>p</td>
<td>3</td>
<td>Li</td>
<td>Lithium</td>
<td>6.941</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Be</td>
<td>Beryllium</td>
<td>9.012</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>d</td>
<td>4</td>
<td>Be</td>
<td>Beryllium</td>
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</tr>
<tr>
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<td>5</td>
<td>d</td>
<td>5</td>
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<td>Magnesium</td>
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<td>6</td>
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<td>d</td>
<td>6</td>
<td>Al</td>
<td>Aluminum</td>
<td>26.982</td>
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<td>Sulfur</td>
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<td>35.453</td>
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<td>f</td>
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<td>f</td>
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<td>f</td>
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<td>Gallium</td>
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<td>25</td>
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<td>f</td>
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<td>31</td>
<td>f</td>
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<td>Yt</td>
<td>Yttrium</td>
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<tr>
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<td>32</td>
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<td>34</td>
<td>34</td>
<td>f</td>
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<td>Mo</td>
<td>Molybdenum</td>
<td>95.94</td>
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<tr>
<td>35</td>
<td>35</td>
<td>f</td>
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<td>f</td>
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<td>Ag</td>
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<td>42</td>
<td>f</td>
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<td>Te</td>
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**Lanthanide Series**

- Ce
- Pr
- Nd
- Pm
- Sm
- Eu
- Gd
- Tb
- Dy
- Ho
- Er
- Tm

**Actinide Series**

- Th
- Pa
- U
- Np
- Pu
- Am
- Cm
- Bk
- Cf
- Es
- Fm
- Md
- No
- Lr

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Mass numbers in parentheses are those of the most stable or most common isotope.

Revised October 15, 2001